

Measurement of production branching ratio after muon nuclear capture reaction of Al and Si isotopes

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Background: Muon nuclear capture is a reaction between a muon and a proton inside a nucleus through weak interactions. This reaction results in the formation of an excited nucleus, which subsequently deexcites by emitting several particles. Examination of the excited state allows for an investigation of the properties of nuclear excitation and particle emission in highly excited nuclei.

Purpose: This study investigates muon nuclear capture of ^{27}Al and $^{28,29,30}\text{Si}$, focusing on determining the absolute production branching ratio (BR) following muon nuclear capture and subsequent particle emissions. By measuring the absolute production BR, we can collect valuable information on the excitation energy distribution of muon nuclear capture.

Methods: Measurements were conducted using the in-beam activation method at two pulsed muon facilities: RIKEN-RAL beamline, ISIS Neutron and Muon Facility at Rutherford Appleton Laboratory and Materials and Life Science Experimental Facility at Japan Proton Accelerator Research Complex. Absolute BRs were determined by measuring the number of muons irradiating the target using a plastic scintillator and the β -delayed γ rays emitted from the produced nuclei using germanium detectors.

Results: The absolute production branching ratios of muon nuclear capture on ^{27}Al and $^{28,29,30}\text{Si}$ were obtained with the highest accuracy to date. Predominant neutron emissions, even-odd atomic number dependence of particle emission probabilities, and influence of the neutron excess were observed. These results were compared with previous measurements and theoretical models and discussed regarding the excitation energy distribution, particle emission mechanism, and nuclear properties, such as resonance in the isovector transition.

Conclusion: This study emphasizes the importance of considering nuclear structure effects, even-odd effects of proton and neutron numbers, neutron excess, nucleon pairing effect, and particle emission mechanisms, in the context of the muon nuclear capture reaction.

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I. INTRODUCTION

A muonic atom is an atomic-bound state comprising a nucleus and a negative muon. After the muon cascades to its 1s atomic orbit, some of the muons are captured by a proton through a weak interaction, a process known as muon nuclear capture. The reaction involving a nucleus with (A, Z) can be expressed as $\mu^- + (A, Z) \rightarrow (A, Z-1)^* + \nu_\mu$, where A and Z represent the mass and atomic numbers of the nucleus,

respectively. The reaction produces an excited $(A, Z-1)$ nucleus owing to the large mass of the muon, which is $105.6 \text{ MeV}/c^2$. Consequently, these highly excited nuclei can emit several particles, such as neutrons, charged particles, and γ rays. Muon nuclear capture is classified as a charge exchange reaction, similarly to negative pion or electron capture. However, unlike pion capture, muon nuclear capture does not involve a strong interaction. Additionally, muon capture can occur in all nuclei owing to the muon's significant mass, whereas electron capture necessitates energy exceeding that of the reaction threshold. Another distinguishing feature of muon nuclear capture is its ability to populate a highly excited

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state with minimal angular momentum transfer. The initial state comprises a static atomic state with a $1s$ muon (spin $1/2$ and zero orbital angular momentum) and a proton within the nucleus (possessing orbital angular momentum). In addition, the muon nuclear capture occurs through a weak interaction, and the highly excited state generated by this process has the potential to provide less model-dependent insight into giant resonance, particularly isovector transition. This complements charge exchange reactions, such as (n, p) reaction. However, muon nuclear capture and particle emission have no established model owing to the lack of experimental data. The distribution of excitation energy cannot be directly measured owing to the challenges associated with measuring the energy of escaped ν_μ . Consequently, our research focuses on the muon nuclear capture reaction and subsequent particle emission, leveraging our current knowledge of nuclear reactions, advanced measurement techniques, and intense muon beams.

The structure of the excitation function, the excitation energy distribution, is reflected in particle emissions. Based on previous measurements of the emitted particle energies, the typical average excitation energy is approximately 10–20 MeV, with the energy reaching up to approximately 100 MeV. However, direct measurement of emitted particles is limited by the particle separation energy and energy deposition within the target material. To overcome this limitation, we have measured the production branching ratio (BR) of the remaining nuclei after particle emission to assess particle emission probabilities. Production BRs after muon nuclear capture have only been measured for a limited number of nuclei because of experimental limitations. Recently, a method referred to as the in-beam activation method capable of measuring short-lived nuclei using the activation method has been developed, and production BRs of muonic palladium isotopes have been comprehensively measured [1]. To accurately measure the absolute BR, the number of stopped muons must be counted. However, quantifying the number of muons within a beam pulse at a high-intensity muon beam facility presents a significant challenge. This study conducted step-by-step measurements to assess the absolute number of stopped muons using a high-intensity muon beam, and the in-beam activation method was extended to measure the absolute production BR.

^{27}Al and $^{28,29,30}\text{Si}$ were selected as the target because they were optimal for measuring the production BR and exploring excited states. ^{28}Si , in particular, was selected owing to the extensive prior research on muon nuclear capture [2]. However, previous measurements in this atomic mass region have been somewhat ambiguous and have displayed discrepancies, warranting a reevaluation. Furthermore, the emission of charged particles, in addition to the neutron emission, is anticipated in this mass region. Recent studies have focused on charged-particle emission [3–6]. The production BR of charged particles is expected to provide insights into the characteristics of particle emission from highly excited nuclei and the emission probabilities of light nuclei. These insights are crucial for understanding the cluster properties within the nucleus. In this study, both natural abundance ($^{\text{nat}}\text{Si}$) and isotopically enriched silicons ($^{28,29,30}\text{Si}$) were measured. Additionally, ^{27}Al was measured owing to its monoisotopic nature and proximity in atomic number to silicon isotopes,

making it ideal for investigating the even-odd effect of atomic number. This mass region facilitates a comprehensive discussion based on both the shell structure and collective properties of the nucleus.

The remainder of this paper is organized as follows. The experimental method and the details of the experiments are described in Secs. II and III, respectively. The analysis procedure is presented in Sec. IV and the result of the deduced BRs for all the targets is summarized in Sec. V. In Sec. VI, the obtained BRs are discussed by comparing them with previous results and model calculations and by considering the systematics with mass and atomic number. Finally, Sec. VII summarizes and concludes the paper.

II. METHOD

In this study, the production BR (b) is defined as

$$b \equiv \frac{N_{\text{prod}}}{N_{\text{cap}}}, \quad (1)$$

where N_{prod} represents the number of produced nuclei and N_{cap} represents the number of muon nuclear capture reactions. N_{prod} was determined using the in-beam activation method, which is a technique developed to measure short-lifetime isotopes with an activation method that utilizes the time structure of the pulse beam [1]. In the in-beam activation method, the β -delayed γ rays originating from the produced nuclei are measured during the interpulse period of a few tens of milliseconds. This enables the measurement of nuclei with lifetimes ranging from a few milliseconds to hours using the activation method. This method was chosen for the present measurement owing to the short lifetimes of most reaction products for silicon isotopes. Throughout this study, the same notation as in Ref. [1] will be used.

N_{cap} is expressed with the total number of muons irradiating the target (N_{muon}), muon nuclear capture probabilities (P_{cap}), and the beam stopping rate in the target (ϵ_{stop}), that is,

$$N_{\text{cap}} = N_{\text{muon}} P_{\text{cap}} \epsilon_{\text{stop}} = \Sigma n_{\text{beam}} P_{\text{cap}} \epsilon_{\text{stop}}, \quad (2)$$

where n_{beam} represents the number of muons in one pulse. ϵ_{stop} was set to be equal to one by adjusting the beam momentum and target thickness as outlined in Sec. III. P_{cap} of aluminum and silicon isotopes were obtained using the following equation:

$$P_{\text{cap}} = \Lambda_{\text{cap}} \tau_{\text{total}}, \quad (3)$$

where Λ_{cap} represents the muon capture rate and τ_{total} represents the lifetime of the muonic atom referred from Ref. [7]. Counting the number of muons in each pulse accurately is generally a challenging task when dealing with a high-intensity beam. Therefore, to obtain the absolute BR with high statistics by measuring n_{beam} , a series of measurements were performed. At first, the absolute BR measurement for dominant reaction products was conducted using a countable low-intensity pulsed muon beam. The beam irradiation number of a high-intensity pulsed muon beam was then calibrated using the measured absolute BR and Eqs. (1) and (2). Finally, the high-statistics BR measurement was conducted using the

calibrated high-intensity beam, especially for isotopes having small production BR.

Experiments were performed at two pulsed muon beam facilities: the absolute BR measurement at the ISIS Neutron and Muon Source at Rutherford Appleton Laboratory (RIKEN-RAL beamlines) [8,9] and the high-statistics BR measurement at the Materials and Life Science Experimental Facility (MLF) at the Japan Proton Accelerator Research Complex (J-PARC) [10]. The intensity of the muon beam at RIKEN-RAL beamlines can be reduced to a countable number and provide less electron contamination. On the other hand, the high-intensity beam at J-PARC provided high statistical data and was ideal for comprehensive measurement, including those involving low-probability branches. Additionally, the high-intensity beam offered a high signal-to-noise (S/N) ratio because the typical background of the activation measurement is natural background radiation.

III. EXPERIMENT

Two experiments were conducted at RIKEN-RAL and J-PARC, which are referred to as the RAL experiment and the J-PARC experiment, respectively, hereinafter. The RAL experiment was conducted at Port 4 of RIKEN-RAL at ISIS. A proton beam of 800 MeV, accelerated in the ISIS RCS synchrotron, irradiated a graphite target at Target Station 1 (TS1) and produced pions. The ISIS synchrotron output has a double-pulse structure with a 50-Hz repetition rate, with four of five pulses directed to TS1. The negative muon beam, resulting from the decay of the negative pions, was then transported and delivered to Port 4. The J-PARC experiment was conducted at the D1 area at MLF, J-PARC. A graphite pion production target was irradiated with a proton beam accelerated to 3 GeV by an RCS synchrotron, operating at an intensity of approximately 730 kW. The negative muon beam was obtained from the decay of pions to the D1 area. The muon beam had a double-pulse structure with a 25-Hz repetition rate.

The schematics of the experimental setups at (a) RAL and (b) J-PARC are shown in Fig. 1. The concept of the setup was consistent in both experiments. The number of muons irradiating targets was counted using a plastic scintillator installed upstream of the target. In the RAL experiment, the plastic scintillator was set approximately 35 mm away from the end of the beam collimator, whereas in the J-PARC experiment, it was placed 30 mm from the end of the beam duct. The plastic scintillator had a thickness of 0.5 mm, collimated by a 10-mm-thick acrylic plate with a diameter of 20 mm. Photomultipliers (PMT) manufactured by Hamamatsu (H11934-100-10 and H11934-100 in the RAL and J-PARC experiments, respectively) were utilized to read out the signal from the plastic scintillator. Two PMTs were employed in the RAL experiment on both sides with an applied voltage of 800 V, whereas in the J-PARC experiment, one PMT was utilized with a voltage of 720 V to prevent output signal saturation.

The activation of five targets was measured, including aluminum and silicon targets with both natural abundance and isotopically enriched compositions ($^{28,29,30}\text{Si}$). The aluminum and natural abundance silicon targets were in the form of metal plates, whereas the enriched targets were in powder and

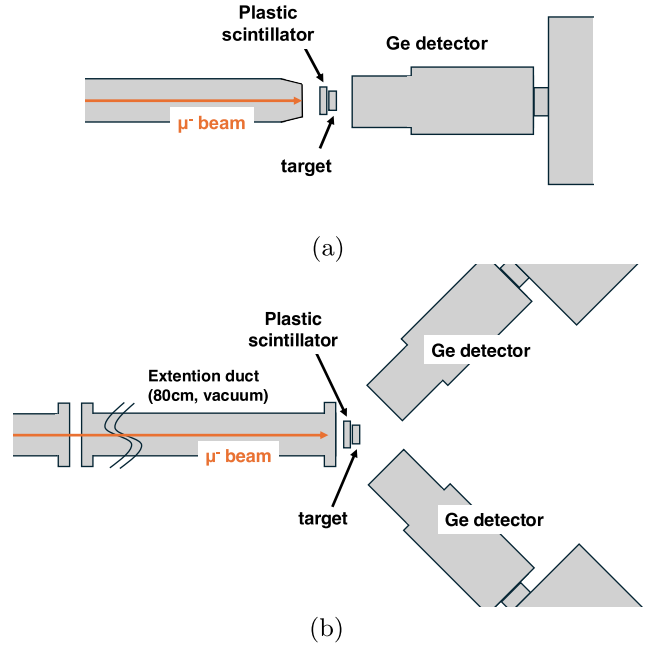


FIG. 1. Schematic of the experimental setup of (a) RAL experiment at Port 4, ISIS, RAL, and (b) J-PARC experiment at D1 area, MLF, J-PARC (not to scale). In the J-PARC experiment, the beam duct was extended by 80 cm, with the detectors set downstream to a μSR spectrometer.

small piece form, as shown in Fig. 2. The enriched targets were encapsulated in acrylic target cases. For the background reference, an empty acrylic case was also irradiated. Details regarding the form, size, and enrichments of each target are listed in Table I. The probability of the muon nuclear capture reaction for each target is also provided in Table I [7]. The targets were positioned 1 mm away from the plastic scintillator to ensure that all muons passing through the plastic scintillator stopped in the target, particularly important for metal plate targets.

The momentum of the muon beam (p) was 36 MeV/c ($\Delta p/p = 4\%$) at RAL and 41 MeV/c ($\Delta p/p = 3\%$) at J-PARC guarantee that all muons stopped at the plate target. A plastic scintillator with a size of $50 \times 50 \times 5 \text{ mm}^3$ was positioned behind the targets prior to the measurement and served as a veto counter to validate that muons did not pass through the target. The isotopically enriched targets were in powder form, so some of the irradiated muons may have

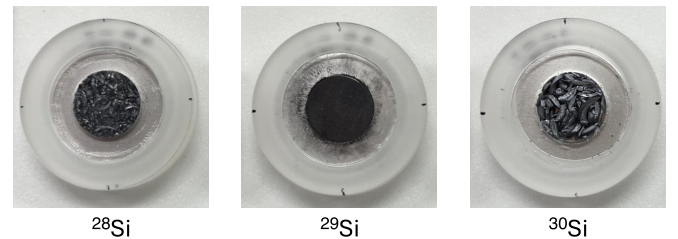


FIG. 2. Photographs of isotopically enriched silicon targets. Target materials were contained in a hole of the acrylic case with a diameter and thickness of 15 and 2.8 mm, respectively.

TABLE I. Summary of the composition and shape information of targets. The probability of the muon nuclear capture reaction (P_{cap}) is listed in the sixth column [7]. Furthermore, the measurement conditions of each target were summarized. Beam irradiation and decay measurement times are listed in the first and second columns of each experiment, with the stopping rate ϵ_{stop} summarized for each target.

Target	Form	Size(mm ³)	Weight(g)	Enrichment	$P_{\text{cap}}(\%)$	RAL			J-PARC		
						Irradiation	Decay	ϵ_{stop}	Irradiation	Decay	ϵ_{stop}
²⁷ Al	Metal plate	50 × 50 × 2	13.6	99+%	60.96(4)	20.0 h	0.3 h	1.0	0.7 h	0.0 h	1.0
^{nat} Si	Metal plate	ϕ50 × 1.96	9.00	–	66.07(5)	8.3 h	0.4 h	1.0	7.3 h	6.7 h	1.0
²⁸ Si	Metal powder	ϕ15 × 2.8	0.500	99.93% ^a	66.41(23)	14.8 h	10.0 h	0.407(14)	4.7 h	0.25 h	0.298(9)
²⁹ Si	Metal powder	ϕ15 × 2.8	0.500	99.25% ^b	64.14(16)	5.4 h	3.0 h	0.652(169)	4.7 h	0.25 h	0.260(63)
³⁰ Si	Small pieces	ϕ15 × 2.8	0.500	99.64% ^c	61.21(12)	23.7 h	29.2 h	0.349(26)	4.5 h	0.25 h	0.261(19)
Empty	Acrylic case	–	–	–	–	4.0 h	0.0 h	–	1.5 h	0.0 h	–

^aThe other composition of ²⁸Si is 0.065% of ²⁹Si and 0.005% of ³⁰Si.

^bThe other composition of ²⁹Si is 0.20% of ²⁸Si and 0.55% of ³⁰Si.

^cThe other composition of ³⁰Si is 0.32% of ²⁸Si and 0.04% of ²⁹Si.

passed through them. Additionally, the radius of the acrylic target case was smaller than that of the plastic scintillator. To determine the ϵ_{stop} of the enriched targets, the natural abundance silicon target was utilized for decomposition analysis in Sec. IV B. The difference in the momentum setting between RAL and J-PARC was attributed to the presence of an additional beam duct in the J-PARC experiment. In the J-PARC experiment, the beam transport line was extended by 80 cm with a vacuum duct after the muon beam passed through the air. The extended beam duct was introduced to place detectors downstream of a μ SR spectrometer in the D1 area. The momentum of the muon beam was degraded with the Kapton foil at both ends of the extended beam duct.

High-purity *p*-type coaxial germanium detectors were used for the decay γ -ray measurement. Details regarding the detector performance and the data acquisition system of the germanium detectors were summarized in Ref. [11]. In the RAL experiment, a Canberra GX5019 was placed at 0° relative to the muon beamline and 58 mm away from the target position. The dynamic range of the measured γ ray obtained with the germanium detector was set to 0.020–2.6 MeV. The typical count rate of the germanium detector was approximately 40–60 counts per second (cps) during beam irradiation and approximately 15 cps without the beam. In the J-PARC experiment, two Canberra-manufactured germanium detectors, GX5019 and GC3018, were utilized. They were positioned at 45° to the muon beamline and 55 mm away from the target. Owing to the higher intensity and larger electron contamination in J-PARC, the germanium detectors could not be set at 0° to the beamline. The dynamic ranges of the germanium detectors were 0.070–10.4 MeV with GX5019 and 0.015–2.1 MeV with GC3018. During beam irradiation, the typical count rate for each germanium detector was approximately 100–200 cps, dropping to 20–30 cps without the beam. To minimize natural background radiation, the detectors were shielded by lead blocks in both experiments.

All the signals from the detectors were acquired using 500-MS/s 14-bit waveform digitizers, CAEN V1730B and V1730SD [12], in both experiments. The firmwares of the digitizer boards were equipped with digital pulse processing for pulse height analysis (DPP-PHA) and DPP for charge integration and pulse shape discriminator, respectively. The

energy and timestamp data from the germanium detectors were processed using DPP-PHA. Additionally, the waveforms of all the PMT signals from plastic scintillators were recorded. A signal from the germanium detectors was acquired and saved in the self-trigger mode, whereas signals from the plastic scintillator were obtained based on an external trigger. The 50- or 25Hz trigger condition from the synchrotron was used as the external trigger in the RAL and J-PARC experiments, respectively, and data were recorded under both beam-on and beam-off. The timing signal from the Cerenkov counter installed near the pion production target was also obtained and served as a precise timing reference for muon beam irradiation in the RAL experiment. In the J-PARC experiment, the RCS kicker signal was used as a precise timing reference instead of a Cherenkov signal.

The measurement conditions for each target are listed in Table I. β -delayed γ rays were measured both during the beam irradiation and after the irradiation (decay measurement) because certain γ -ray energy peaks from the long-lived isotopes produced demonstrate a higher S/N ratio in the off-beam condition.

IV. ANALYSIS

Figure 3 shows the segment of the γ -ray energy spectra of each target obtained with a germanium detector at the RAL experiment. The activation method utilized these energy spectra to calculate the absolute BRs by inferring the muon irradiation number from a plastic scintillator signal. In this section, data evaluation, activation analysis, absolute production BR analysis using a low-intensity beam, and the calibration method of the plastic scintillator with a high-intensity beam are described.

A. Activation analysis and data evaluation

The number of reaction products can be determined through β -delayed γ -ray spectroscopy, leveraging the well-established β -decay schemes of the reaction products. The number of produced nuclei (N_{prod}) was determined using the in-beam activation method, and is expressed as follows:

$$N_{\text{prod}} = \frac{N_{\gamma}/(\epsilon_{\gamma}\epsilon_{\text{LT}})}{P_{\text{decay}}I_{\gamma}} - N_{\text{decay}}, \quad (4)$$

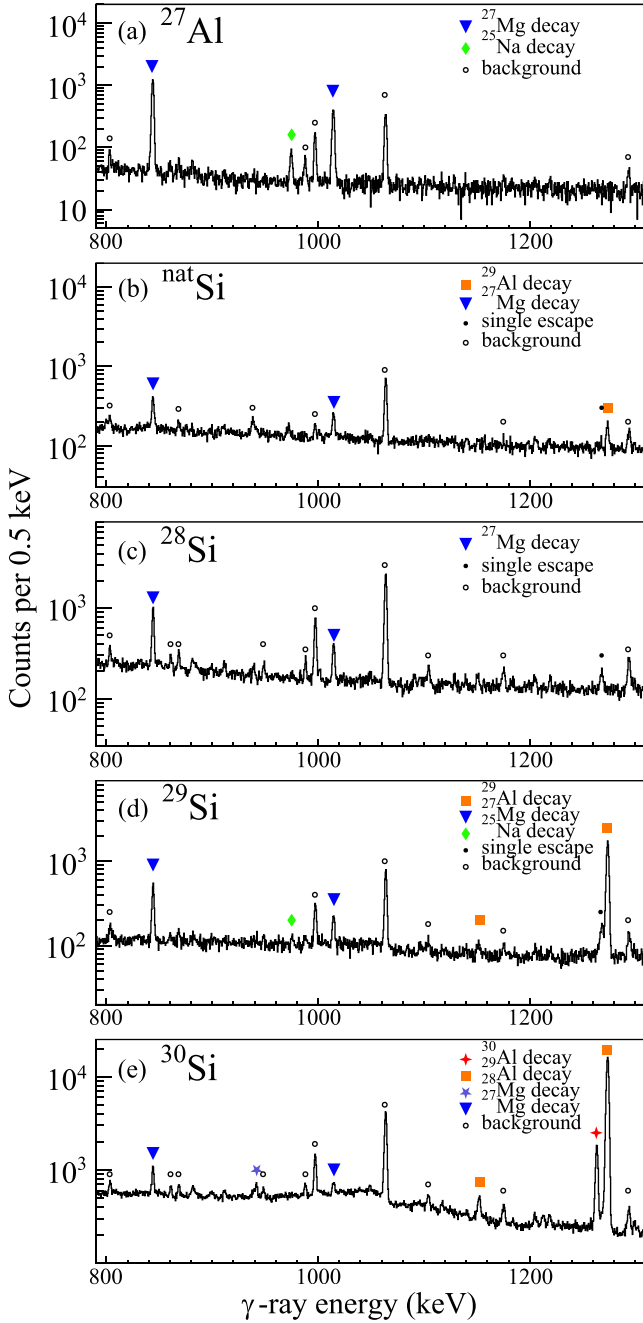


FIG. 3. Part of the γ -ray energy spectra of ^{27}Al , $^{\text{nat}}\text{Si}$, and $^{28,29,30}\text{Si}$ target obtained with a germanium detector at the RAL experiment. β -delayed γ -ray energy peaks are denoted with each symbol, background peaks are denoted with open circles, and peaks originating from single escapes of other peaks are denoted with filled circles.

where N_γ represents the number of the detected β -delayed γ ray, ϵ_γ represents the γ -ray detection efficiency of the detectors, ϵ_{LT} represents the analysis live-time ratio, P_{decay} represents the decay probability during the measurement period, and I_γ represents the γ -ray intensity per decay of the subject nuclei. In this study, N_{prod} is defined as the direct production number, obtained by subtracting de-

cay components from β -decay of other nuclei and isomeric decay (N_{decay}).

The N_γ values were determined based on the measured energy peaks of β -delayed γ ray, some of which are indicated by closed symbols in Fig. 3. The identification of these γ rays was achieved through their energy, with additional confirmation provided by half-life for some γ rays. The energy calibration of germanium detectors was conducted with standard γ -ray sources. To account for any shifts in energy peaks obtained during the measurement, a correction for gain drift was applied using typical γ -ray energy peaks assuming the linear gain drift [11]. Additionally, potential overlaps with single and double escape peaks from the higher energy peaks were examined for all the observed peaks. The background component of each γ -ray energy peak was subtracted, if present. The sources of this background included environmental background and beam irradiation on materials other than the target. For example, the creation of background ^{28}Al through the neutron capture reaction, $^{27}\text{Al}(n, \gamma)^{28}\text{Al}$ was attributed to neutron presence in the muon beamlines and the aluminum components of the germanium detector's cryostat case. In the J-PARC experiment, particular attention was paid to the background contribution from beam irradiation on the stainless steel beam duct, which contained silicon approximately 1%. The environmental background level was assessed through background measurements conducted without beam irradiation and then normalized based on the measurement time. Additionally, the background component associated with beam irradiation was determined through empty target measurements and normalized based on the muon irradiation number during each measurement.

The peak detection efficiency of the germanium detector, ϵ_γ , was calibrated using standard γ -ray sources in the RAL experiment and evaluated through GEANT4 simulation [13–15] for the J-PARC experiment. The uncertainty of the GEANT4 simulation was comparable with that of the standard γ -ray sources, approximately 3% [11]. Furthermore, the impact of self-absorption of emitted γ ray within the target and target case was estimated using the GEANT4 simulation. To validate the accuracy of the self-absorption estimation using GEANT4, the efficiency was compared with and without a 5.0-mm-thick acrylic plate placed in front of the standard γ -ray sources.

Owing to the close proximity of the detector to the target, some x rays and γ rays were detected simultaneously at the prompt timing of the beam arrival. This resulted in the signals from the germanium detector becoming saturated for a few milliseconds after beam irradiation. Therefore, signals within 0.17 ms (RAL experiment) and 2.0 ms (J-PARC experiment) following beam irradiation were excluded from the analysis, with the effect being accounted for through the use of a live-time ratio, ϵ_{LT} , in the analysis process.

P_{decay} was calculated using the half-life of the produced nuclei ($T_{1/2} = \ln(2)/\lambda$) and the muon irradiation number as follows:

$$P_{\text{decay}} \equiv \frac{\int \lambda n_{\text{nucl}}(t) dt}{N_{\text{muon}}}. \quad (5)$$

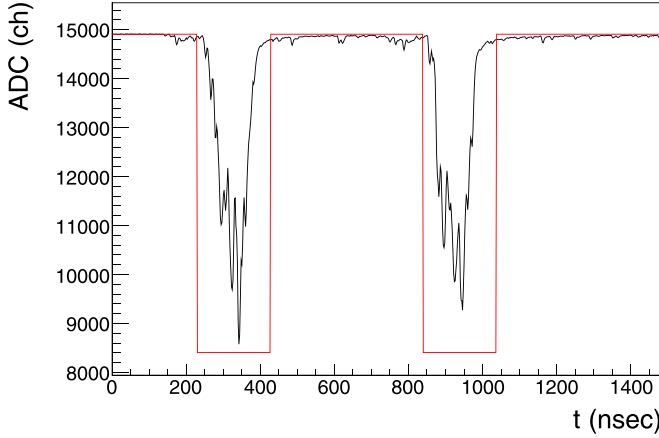


FIG. 4. Raw signal of the plastic scintillator from PMT in one pulse obtained at J-PARC. The time distance from the kicker signal is shown on the horizontal axis. The red line represents the charge integral gate of the muon irradiation. The gate width was chosen to be 150 and 200 ns in the RAL and J-PARC experiments, respectively.

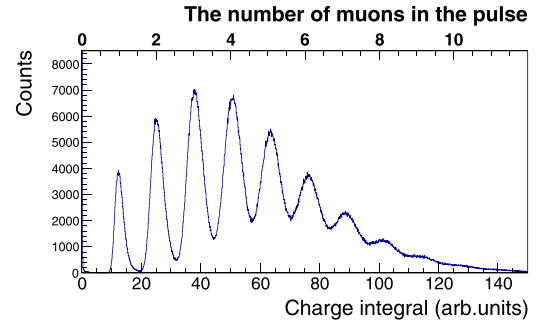
Here the numerator represents the total decay number during the measurement period assuming all the irradiated muon produced the given nucleus ($P_{\text{cap}}\epsilon_{\text{stop}}b = 1$) [1], where $n_{\text{nucl}}(t)$ represents the number of radioactive reaction products at time t while assuming $P_{\text{cap}}\epsilon_{\text{stop}}b = 1$ as well. The half-life of the produced nuclei and I_γ were obtained from Evaluated and Compiled Nuclear Structure Data (ENSDF) [16–27].

The N_{prod} was calculated with Eq. (4) using N_γ with and without beam irradiation. The subtraction of N_{decay} was conducted by solving the Bateman equation [1,28]. The direct production number of isomeric states was determined independently from the ground-state production.

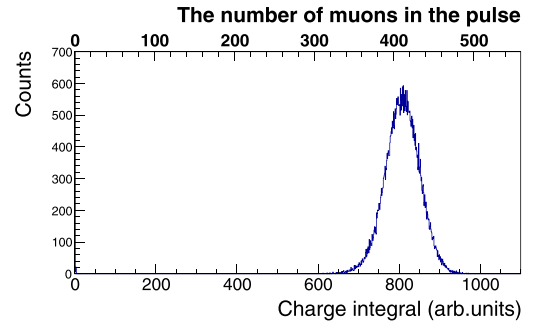
B. Number of muons irradiating the target

The number of muons irradiating the target [N_{muon} in Eq. (2)] was measured by utilizing a plastic scintillator. The energy deposit of the muons through the plastic scintillator is proportional to the number of muons irradiating the target. The raw signal from the PMT in one pulse obtained at J-PARC is shown in Fig. 4. The smaller signal detected prior to muon irradiation corresponds to electron contamination in the beam. The signals from the plastic scintillator underwent waveform analysis. The charge integral of the signals was determined by integrating the signal height within the muon irradiation timing gate indicated by the red line. The charge integral gate for muon irradiation was carefully selected to ensure that all muons within the pulse were included while minimizing electron contamination. In the RAL experiment, the gate width was set at 150 ns with a 330-ns distance, whereas in the J-PARC experiment, it was at 200 ns with a 610-ns distance. The spectrum of the charge integral obtained through waveform analysis is shown in Fig. 5.

The muon beam intensity at RAL was meticulously adjusted to ensure precise quantification of the muon irradiation during the experiment. As shown in Fig. 5(a), the peaks of the charge integral of the plastic scintillator were clearly distinguished, indicating the number of muons present in each



(a)



(b)

FIG. 5. Spectra of the charge integral obtained by utilizing the plastic scintillator at (a) RAL and (b) J-PARC. The bottom horizontal axis represents the charge integral with arbitrary units, whereas the vertical axis represents the count. The upper horizontal axis represents the absolute muon number in the pulse. The momentum of the muon beam was 35 MeV/c at RAL and 38 MeV/c at J-PARC, respectively. The beam intensity was reduced to countable numbers at RAL.

pulse at RAL. The charge integral of the plastic scintillator was calibrated to the number of muons by fitting each peak in the histogram. Utilizing the derived muon count in each pulse, the absolute production BR was calculated using the Eqs. (1), (2), and (4).

In this experiment, the enriched silicon targets were in powder form, as shown in Fig. 2. Some muons were not fully stopped in the target, $\epsilon_{\text{stop}} \neq 1$, for these powder targets. Therefore, the determined BR values are relative and expressed as follows: $b_{\text{rel}} = \epsilon_{\text{stop}} b$. The absolute BR of each enriched target can be determined by decomposing utilizing the result of the natural abundance silicon target as

$$P_{\text{cap}}^{\text{nat Si}} b_i^{\text{nat Si}} = \sum_j A^j P_{\text{cap}}^j \frac{1}{\epsilon_{\text{stop}}^j} b_i^j, \quad (6)$$

where i represents produced nuclei, j corresponds to each isotope ($j = {}^{28,29,30}\text{Si}$), and A^j represents the abundance of each isotope ($A^{28}\text{Si} = 0.9223$, $A^{29}\text{Si} = 0.0467$, $A^{30}\text{Si} = 0.0310$). ϵ_{stop}^j were calculated using least square method with Eq. (6) and production BRs of four produced nuclei, ${}^{30}\text{Al}$, ${}^{29}\text{Al}$, ${}^{28}\text{Al}$, and ${}^{27}\text{Mg}$ from ${}^{\text{nat Si}}$ and ${}^{28,29,30}\text{Si}$. Furthermore, for the enriched targets ${}^{28,29,30}\text{Si}$, the impurity of the target listed in Table I was simultaneously considered in the decomposition process. The

determined stopping rates of the various targets are listed in Table I. The discrepancy in the ϵ_{stop} values between the two experiments may be attributed to variations in the momentum bite and the focus of the beam spot. Through this analysis, the absolute BRs of aluminum and silicon isotopes were determined using measurable muon irradiation at RAL.

As the beam intensity becomes higher, the spectrum of the charge integral of the plastic scintillator becomes continuous, as shown in Fig. 5(b). This figure displays the spectrum of the charge integral obtained with the high-intensity muon beam containing approximately 400 particles per pulse (ppp) at J-PARC. The linearity, ranging from the countable muons less than approximately 10 ppp to high-intensity muon beams, was not guaranteed owing to the potential quenching effect of the plastic scintillator by a high-density pulse beam. Therefore, the calibration of the charge integral to the muon number was conducted in the J-PARC experiment.

The absolute BR of the muon nuclear capture reaction of the ^{27}Al target was utilized for this calibration. Assuming linearity between the charge integral of the PMT signal and the number of muons passing through the plastic scintillator within the range of beam fluctuation, the number of muons can be expressed as $n_{\text{beam}} = A \times Q$, where Q represents the charge integral of the PMT signals and A represents the calibration parameter to convert Q to the number of muons. The calibration parameter A was determined through in-beam activation measurement of ^{27}Al using Eqs. (1) and (2), and the absolute BR of the $^{27}\text{Al}(\mu^-, \nu_\mu)^{27}\text{Mg}$ reaction, which was 9.90(33)% as determined in the RAL experiment. Notably, the effect of the electron contamination in the muon charge integral gate at J-PARC was included in the calibration parameter Q .

After the calibration process was conducted, the absolute production BR was calculated using the same methodology as in the RAL experiment. This approach enabled a high statistical absolute BR measurement with the high-intensity pulsed muon beam at J-PARC.

C. Uncertainty

The following items were considered as uncertainty: statistical uncertainty (including uncertainties of the background component when it was subtracted), uncertainty in I_γ , uncertainty in half-lives ($T_{1/2}$), uncertainty in the peak detection efficiency of the germanium detectors (ϵ_γ), systematic uncertainty in the number of muons irradiating the target (N_{muon}), uncertainty in the stopping rate (ϵ_{stop}), and uncertainty in the capture probability (P_{cap}).

The uncertainty in I_γ and half-life value was cited from the ENSDF database, and the uncertainty in capture rates was determined based on the uncertainty in the referenced lifetime of muonic atoms [7]. The uncertainties stemming from the lifetime and muon capture rate were negligible compared with other uncertainties. For I_γ , the uncertainties in the relative intensity of each energy γ rays ($\Delta I_\gamma^{\text{rel}}$) and absolute intensity ($\Delta I_\gamma^{\text{abs}}$) were treated separately, following the approach outlined in the Ref. [1]. $\Delta I_\gamma^{\text{rel}}$ and $\Delta I_\gamma^{\text{abs}}$ were sourced from the uncertainty of each γ -ray column and that of the normalization factor in the ENSDF database, respectively.

These uncertainties were distinctly labeled in Tables II–VI. The uncertainty of the stopping rate was determined during the decomposition process.

In the RAL experiment, as the absolute BR value is determined using the yield of γ ray and the measured number of muons, the uncertainty in the efficiency of the germanium detectors is considered as an absolute uncertainty. The uncertainty in the number of muon irradiations in the RAL experiment was negligible owing to the clear separation of peaks, as shown in Fig. 5(a). The peak detection efficiency of the germanium detectors had a systematic uncertainty of 3%, originating from the uncertainty in the activities of standard γ -ray sources. At J-PARC, only the relative values for each target were measured and normalized using the BR of $^{27}\text{Al}(\mu^-, \nu_\mu)^{27}\text{Mg}$ during the calibration process. The uncertainty in the relative efficiency of the germanium detectors was deemed negligible; hence, the germanium efficiency uncertainty was not considered independently. The systematic uncertainty in the number of muons in the J-PARC experiment was attributed to the calibration uncertainty of the plastic scintillator, primarily resulting from the uncertainty in the absolute BR of ^{27}Mg produced from muonic ^{27}Al . The uncertainty in the BR of ^{27}Mg from ^{27}Al was dominated by the germanium efficiency uncertainty at the RAL experiment.

V. RESULT

The absolute BR results of produced nuclei following muon nuclear capture and subsequent particle emissions are listed in Tables II–VI for ^{27}Al , $^{\text{nat}}\text{Si}$, and $^{28,29,30}\text{Si}$, respectively. The tables include information on observed γ rays and BR derived from each γ ray yield from two experiments, as well as compiled values. The weighted average values of BR for each residual nucleus are provided below the results for each γ ray, accounting for decay components from the mother nuclei and isomer, and the impurities in enriched targets. The first parentheses in the uncertainty of weighted average and compiled value represent the relative uncertainty, including statistical uncertainty, uncertainties from I_γ (both relative and absolute), and the half-life of the nuclei. The second parentheses account for the uncertainties associated with the efficiency of the germanium detector, muon capture rate, and the number of muons detected by the plastic scintillator while excluding the relative uncertainty. For enriched silicon targets, the uncertainty in the stopping rate is also included in the second set of parentheses. The uncertainty in the first parenthesis corresponds to the relative uncertainty of each reaction product in the target, transitioning to absolute uncertainty when the uncertainty in the second set of parentheses is added. Furthermore, the upper limits of BRs of some reaction products were presented. For undetected nuclei, the upper limit evaluated with the detection limit was provided. In the case of two candidates for the origin of detected γ ray, the upper limit was provided along with uncertainties in the final column.

The reaction channels were denoted based on the emitted number of neutrons and protons, hereinafter; for example, the production of ^{27}Mg and ^{23}Ne from muon nuclear

TABLE II. Absolute production BR of each isotope produced from muonic ^{27}Al . $P_{\text{cap}} = 60.96(4)\%$ was utilized as the muon capture probability. The parent nucleus of the decay (nucleus), spin-parity of the decaying state (state), decay mode (decay), half-life ($T_{1/2}$), γ -ray energy (E_γ), γ -ray intensity (I_γ), BR determined using each γ -ray intensity from the RAL (b_γ^{RAL}) and J-PARC ($b_\gamma^{\text{J-PARC}}$) experiments are listed in the table. The weighted average values of BR of each experiment are listed below the result of each γ -ray peak, with the compiled BR for each decaying state (b) listed in the last column. The decay properties are obtained from ENSDF [16–27].

Nucleus	State	Decay	$T_{1/2}$	E_γ (keV)	I_γ (%) ^b	b_γ^{RAL} (%) ^a	$b_\gamma^{\text{J-PARC}}$ (%) ^a	b (%) ^a
^{27}Mg	$1/2^+$	β^-	9.458(12) min	170.82	0.860(20)	—	— ^g	
				843.76	71.800(20)	9.97(17)	— ^g	
				1014.52	28.200(20)	9.65(32)	— ^g	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.11\%$	9.90(15)(30)	— ^g	9.90(15)(30)
^{23}Mg	$3/2^+$	ϵ	11.3046(45) s	440.5	7.85(11)	3.02(41)	3.42(26)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	3.02(41)(11)	3.42(26)(13)	<3.31(22)(12) ^h
^{26}Na	3^+	β^-	1.07128(25) s	1808.71	99.08(50) ^d	0.79(7)	0.78(7)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.03\%$	0.79(7)(3)	0.78(7)(3)	0.783(50)(28)
^{25}Na	$5/2^+$	β^-	59.1(6) s	389.710	12.68(22)	2.52(21)	2.55(8)	
				585.028	13.00(18)	—	2.39(10)	
				974.742	14.95(22)	2.90(24)	2.49(10)	
				1611.716	9.48(14)	—	2.57(15)	
^{24}Na	4^+	β^-	14.956(3) h	1368.625	$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 5.4\%$	2.68(26)(10)	2.50(16)(9)	2.52(16)(9) ⁱ
					99.994(2)	0.67(18)	—	
	1^+	IT	20.18(10) ms	472.2023	$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	0.67(18)(2) ^c		0.67(18)(2)
					99.95(5) ^e	0.937(37)	1.005(27)	
^{25}Ne	$1/2^+$	β^-	602(8) ms	89.53	$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.0\%$	0.937(37)(34)	1.005(27)(37)	0.980(22)(35) ^j
					95.4(8)	<0.074	<0.035	
^{24}Ne	0^+	β^-	3.38(2) min	472.2023	100.0(2)		<0.275	
					874.420	7.9(2)	<0.255	
^{23}Ne	$5/2^+$	β^-	37.25(10) s	440.5	32.9(10)	0.72(10)	0.83(6)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	0.72(10)(3)	0.83(7)(3)	<0.80(6)(3) ^h
^{21}Na	$3/2^+$	β^-	22.49(4) s	350.725	5.07(13)	1.7(7)	—	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	1.70(66)(6)		<1.70(66)(6) ^k
^{21}F	$5/2^+$	β^-	4.158(20) s	350.725	89.6(18) ^d	0.096(37)	—	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 3.32\%$	0.096(37)(3)		<0.096(37)(3) ^k
^{20}Na	2^+	ϵ	447.9(23) ms	1633.602	79.3(11)	—	0.128(26)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$		0.128(26)(5)	<0.128(26)(5) ^l
^{20}F	2^+	β^-	11.163(8) s	1633.602	99.9995(10) ^f	—	0.102(21)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$		0.102(21)(4)	<0.102(21)(4) ^l
^{19}O	$5/2^+$	β^-	26.88(5) s	1356.843	50.4(11)	<0.037	<0.073	

^aThe uncertainties in the b_γ^{RAL} and $b_\gamma^{\text{J-PARC}}$ columns in each γ -ray row include the statistical uncertainty from the γ -ray peak count and uncertainty from I_γ^{rel} . The first set of parentheses in the weighted average row indicates the relative uncertainty, encompassing the statistical uncertainty, uncertainties from I_γ (relative and absolute), and the lifetime of the nuclei. The second set of parentheses includes the uncertainties of the efficiency of the germanium detector, muon capture rate, and number of muons counted with the plastic scintillator, excluding the relative uncertainty. The uncertainties from the lifetime and muon capture rate were negligible compared with other uncertainties.

^b I_γ uncertainty includes both $\Delta I_\gamma^{\text{rel}}$ and $\Delta I_\gamma^{\text{abs}}$ for the nuclei denoted as $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$

^cDecay component from the mother nuclei was subtracted.

^d $\Delta I_\gamma^{\text{rel}}$ of these peaks are not provided in the ENSDF database and were estimated from other ΔI_γ .

^e $\Delta I_\gamma^{\text{rel}}$ is not provided in the ENSDF database. The uncertainty originating from the probability of beta decay from ^{24m}Na (0.05%) was estimated as 0.05%.

^f $\Delta I_\gamma^{\text{rel}}$ of this peak is not provided in the ENSDF database and was estimated from the probability of the beta decay to the ground state.

^gBR of ^{27}Mg was utilized as the calibration reference in the analysis of the J-PARC experiment.

^hA peak at 440.5 keV originated from ^{23}Mg and ^{23}Ne . The upper limit of these isotopes could not be restricted by the other energy peaks. The production BR of these isotopes both represent the upper limit.

ⁱThe lower limit of ^{25}Na is 2.46% calculated from the upper limit of ^{25}Ne .

^jA peak at 472.2 keV originated from ^{24m}Na and ^{24}Ne . The upper limit of ^{24}Ne is estimated from the statistics during the decay measurement after stopping the beam irradiation, with the upper limit of the nondetected peak at 874.41 keV. The lower limit of ^{24m}Na is 0.679% calculated from the upper limit of ^{24}Ne .

^kA peak at 350.7 keV could originate from ^{21}Na and ^{21}F . The background component was subtracted since the peak was also observed in the empty target measurement.

^lA peak at 1633.6 keV could originate from ^{20}Na and ^{20}F . The upper limit of these isotopes could not be restricted by the other energy peaks. The production BR of these isotopes both represent the upper limit.

TABLE III. Absolute production BR of each isotope produced from muonic natural abundance Si. $P_{\text{cap}} = 66.07(5)\%$ was utilized as the muon capture probability. The same notations as those in Table II were utilized.

Nucleus	State	Decay	$T_{1/2}$	E_γ (keV)	I_γ (%) ^b	b_γ^{RAL} (%) ^a	$b_\gamma^{\text{J-PARC}}$ (%) ^a	b (%) ^a
³⁰ Al	3 ⁺	β^-	3.62(6) s	3498.33	32.6(14) ^d	—	0.396(27)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$		0.396(28)(15)	0.396(28)(15)
²⁹ Al	5/2 ⁺	β^-	6.56(6) min	1152.57	1.031(27)	—	2.80(50)	
				1273.36	91.3(9) ^e	—	2.56(6)	
				2028.09	3.514(27)	—	2.62(18)	
				2425.73	5.23(6)	—	2.78(15)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.07\%$	—	2.57(12)(9) ^c	2.57(12)(9)
²⁸ Al	3 ⁺	β^-	2.245(2) min	1778.987	100(0)	19.69(45)	20.37(22)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.00\%$	19.67(45)(59) ^c	20.33(22)(75) ^c	20.08(28)(60)
²⁹ Mg	3/2 ⁺	β^-	1.30(12) s			—	—	0.019(7) ⁱ
²⁸ Mg	0 ⁺	β^-	20.915(9) h			—	—	0.125(27) ⁱ
²⁷ Mg	1/2 ⁺	β^-	9.458(12) min	843.76	71.800(20)	2.95(17)	3.01(24)	
				1014.52	28.200(20)	3.03(37)	2.75(7)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.11\%$	2.97(16)(9)	2.89(19)(11)	2.94(13)(9) ^j
²⁶ Na	3 ⁺	β^-	1.07128(25) s	1808.71	99.08(50) ^e	<0.388		
				1128.89	5.88(3)		<0.287	
²⁵ Na	5/2 ⁺	β^-	59.1(6) s	389.710	12.68(22)	—	0.207(31) ^h	
				585.028	13.00(18)	—	0.355(40)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 5.4\%$		0.355(44)(13)	0.355(44)(13) ^k
²⁴ Na	4 ⁺	β^-	14.956(3) h	1368.625	99.994(2)	—	—	0.52(10) ⁱ
	1 ⁺	IT	20.18(10) ms	472.2023	99.95(5) ^f	1.21(10)	1.158(27)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.00\%$	1.21(10)(4)	1.158(27)(42)	1.164(32)(35) ^l
²⁵ Ne	1/2 ⁺	β^-	602(8) ms	89.53	95.4(8)	<0.48	<0.074	
²⁴ Ne	0 ⁺	β^-	3.38(2) min	472.2023	100.0(2)		<0.173	
²⁰ Na	2 ⁺	ϵ	447.9(23) ms	1633.602	79.3(11)	—	0.119(9)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$		0.119(9)(4)	<0.119(9)(4) ^m
²⁰ F	2 ⁺	β^-	11.163(8) s	1633.602	99.9995(10) ^g	—	0.094(7)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$		0.094(7)(3)	<0.094(7)(3) ^m

^aThe notation of the uncertainties is the same as that in Table II.

^b I_γ uncertainty includes both $\Delta I_\gamma^{\text{rel}}$ and $\Delta I_\gamma^{\text{abs}}$ for the nuclei denoted as $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$

^cThe decay component from the mother nuclei was subtracted.

^dAbsolute I_γ was not provided in the ENSDF and was calculated from the intensity of beta decay to each energy level and the probability of γ -ray transition from each state [29–31]. The uncertainties encompassed absolute uncertainty.

^e $\Delta I_\gamma^{\text{rel}}$ of these peaks are not provided in the ENSDF database and were estimated from other ΔI_γ .

^f $\Delta I_\gamma^{\text{rel}}$ is not provided in the ENSDF database. The uncertainty originating from the probability of beta decay from ^{24m}Na (0.05%) was estimated as 0.05%.

^g $\Delta I_\gamma^{\text{rel}}$ of this peak is not provided in the ENSDF database and was estimated from the probability of the beta decay to the ground state.

^hThe BR determined with 389.7 keV was minimal compared with the result of the other peaks obtained at J-PARC. This result was excluded from the weighted average value.

ⁱThe BRs of these nuclei were calculated from the result of enriched silicon targets.

^jThe lower limit of ²⁷Mg is 2.93% calculated from the upper limit of ²⁷Na (0.01%), calculated using the result of ^{29,30}Si.

^kThe lower limit of ²⁵Na is 0.280% calculated from the upper limit of ²⁵Ne.

^lThe peak at 472.2 keV originated from ^{24m}Na and ²⁴Ne. The upper limit of ²⁴Ne was estimated from the statistics obtained during the decay measurement after stopping the beam irradiation. The lower limit of ^{24m}Na was 0.998% calculated from the upper limit of ²⁴Ne.

^mA peak at 1633.6 keV could originate from ²⁰Na and ²⁰F. The upper limit of these isotopes could not be limited by the other energy peaks. The production BR of these isotopes both represent the upper limits.

capture of ²⁷Al would be denoted as $0n0p$ and $2n2p$ channels, respectively. Notably, the activation method utilized cannot differentiate between the types of emitted particles that result in the formation of identical residual nuclei, such as processes involving the emission of one neutron and one proton, as well as deuteron emissions processes.

From muonic ²⁷Al, no particle emission ($0n0p$, ²⁷Mg), several neutrons and one proton emission ($xn1p$, ^{26,25,24}Na) were detected. The two-neutron two-proton emission ($2n2p$, ²³Ne) channels may be detected as well, although the detected energy peak could potentially originating from the ²³Mg ($4n0p$ channel). For ^{28,29,30}Si, neutron emission channels (^{30,29,28}Al), one proton emission channels (^{29,28,27}Mg),

TABLE IV. Absolute production BR of each isotope produced from muonic ^{28}Si . $P_{\text{cap}} = 66.41(23)\%$ was utilized. The stopping rates were 0.407(14)% and 0.298(9)% in the RAL and J-PARC experiments, respectively. The same notations as those in Table II were utilized.

Nucleus	State	Decay	$T_{1/2}$	E_γ (keV)	I_γ (%) ^b	b_γ^{RAL} (%) ^a	$b_\gamma^{\text{J-PARC}}$ (%) ^a	b (%) ^a
^{28}Al	3^+	β^-	2.245(2) min	1778.987	100 $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.0\%$	18.94(19) 18.92(20)(94) ^c	18.93(10) 18.91(10)(92) ^c	18.92(10)(91)
^{27}Mg	$1/2^+$	β^-	9.458(12) min	843.76 1014.52	71.800(20) 28.200(20) $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.11\%$	2.82(6) 3.06(14) 2.86(6)(14) ^c	2.93(10) 2.84(9) 2.88(7)(14) ^c	2.869(45)(137)
^{26}Na	3^+	β^-	1.07128(25) s	1808.71 1128.89	99.08(50) ^e 5.88(3)	<0.185	<1.17	
^{25}Na	$5/2^+$	β^-	59.1(6) s	585.028	13.00(18) $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 5.4\%$	0.24(17) 0.24(17)(1) ^c	—	0.24(17)(1) ^b
^{24}Na	4^+	β^-	14.956(3) h	1368.625	99.994(2) $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	0.53(10) 0.53(10)(3) ^{c,d}	—	0.53(10)(3)
	1^+	IT	20.18(10) ms	472.2023	99.95(5) ^f $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.0\%$	1.182(36) 1.182(36)(59) ^c	1.134(48) 1.134(49)(55) ^c	1.161(29)(56) ⁱ
^{25}Ne	$1/2^+$	β^-	602(8) ms	89.53	95.4(8)	<0.244	<0.172	<0.080 ^j
^{24}Ne	0^+	β^-	3.38(2) min	472.2023 874.420	100.0(2) 7.9(2)	<0.568	<0.563	<0.187 ^{ij}
^{20}Na	2^+	ϵ	447.9(23) ms	1633.602	79.3(11) $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	0.144(47) 0.144(47)(7) ^c	0.148(32) 0.148(32)(7) ^c	<0.147(26)(7) ^k
^{20}F	2^+	β^-	11.163(8) s	1633.602	99.9995(10) ^g $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	0.114(37) 0.114(37)(6) ^c	0.117(24) 0.117(24)(6) ^c	<0.116(20)(6) ^k

^aThe notation of the uncertainties is the same as that in Table II. The uncertainty of the stopping rate was included in the uncertainty in the second parenthesis.

^b I_γ uncertainty includes both $\Delta I_\gamma^{\text{rel}}$ and $\Delta I_\gamma^{\text{abs}}$ for the nuclei denoted as $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$

^cThe impurity of the enriched target (Table I) was taken into account with the decomposition process.

^dThe decay component from the mother nuclei was subtracted.

^e $\Delta I_\gamma^{\text{rel}}$ of the peak is not provided in the ENSDF database and was estimated from other ΔI_γ .

^f $\Delta I_\gamma^{\text{rel}}$ is not provided in the ENSDF database. The uncertainty originating from the probability of beta decay from ^{24m}Na (0.05%) was estimated as 0.05%.

^g $\Delta I_\gamma^{\text{rel}}$ of this peak is not provided in the ENSDF database and was estimated from the probability of the beta decay to the ground state.

^hThe lower limit of ^{25}Na is 0.163% calculated from the upper limit of ^{25}Ne .

ⁱA peak at 472.2 keV originated from ^{24m}Na and ^{24}Ne . The upper limit of ^{24}Ne was estimated from the statistics during the decay measurement after stopping the beam irradiation, with the upper limit of the non-detected peak at 874.41 keV. The lower limit of ^{24m}Na is 0.993% calculated from the upper limit of ^{24}Ne .

^jThe upper limit of $^{24,25}\text{Ne}$ was evaluated from the result of natural abundance silicon.

^kA peak at 1633.6 keV could originate from ^{20}Na and ^{20}F . The upper limit of these isotopes could not be restricted by the other energy peaks. The production BR of these isotopes both represent the upper limit.

two protons emission channels ($^{26,25,24}\text{Na}$) were detected. Reaction channels not listed in the table were not detected, either because they were stable nuclei or below the detection limit. Notably, the detection limit for in-beam activation measurement varies depending on the produced nuclei because the detection limit depends not only on the production BR but also on the strength of I_γ of the decay γ ray, efficiency of the germanium detector at the energy, lifetime of the produced nuclei, and measurement time.

The decay components from the mother nuclei, which decayed into the same unstable nuclei as the produced nuclei from the muon nuclear capture, were evaluated if the BR of the mother nuclei was detected. For example, the decay component from the ^{28}Mg in the measured BR of ^{28}Al was subtracted in the analysis of silicon targets. In cases where the BRs of the mother nuclei were not detected, the lower limits of

the daughter nuclei were calculated based on the upper limits of the mother nuclei and are detailed in footnotes. For the isotopically enriched targets, the component originating from the contamination of the other isotopes present in the target was calculated and considered during the decomposition process.

Large uncertainty was observed in the BR of $^{29,30}\text{Si}$ owing to the large uncertainty of ϵ_{stop} , as listed in Table I. Decomposition was conducted using natural abundance silicon, which had limited statistics for nuclei produced from $^{29,30}\text{Si}$ in a $^{\text{nat}}\text{Si}$ target owing to the predominance of ^{28}Si in natural abundance silicon. For example, the values of $b_{^{30}\text{Al}}$ and $b_{^{29}\text{Al}}$ for $^{\text{nat}}\text{Si}$ were 0.396(31)% and 2.57(15)%, respectively, corresponding to relative uncertainties of 8% and 6%, primarily attributed to the statistical uncertainty. The determination of $\epsilon_{\text{stop}}^{^{30}\text{Si}}$ was primarily based on the absolute BR of ^{30}Al , as it can only be produced from the muon nuclear capture of ^{30}Si in $^{\text{nat}}\text{Si}$. Therefore, the 8% uncertainty of $b_{^{30}\text{Al}}$ of $^{\text{nat}}\text{Si}$ propagated

TABLE V. Absolute production BR of each isotope produced from muonic ^{29}Si . $P_{\text{cap}} = 64.14(16)\%$ was utilized. The stopping rates were 0.652(169)% and 0.260(63)% in the RAL and J-PARC experiments, respectively. The same notations as those in Table II were utilized.

Nucleus	State	Decay	$T_{1/2}$	E_γ (keV)	I_γ (%) ^b	b_γ^{RAL} (%) ^a	$b_\gamma^{\text{J-PARC}}$ (%) ^a	b (%) ^a
^{29}Al	$5/2^+$	β^-	6.56(6) min	1152.57 2028.09 2425.73	1.031(27) 3.514(27) 5.23(6) $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.07\%$	20.(6) 13.8(14) 15.3(11) 14.6(9)(39) ^c	15.2(24) 15.7(8) 15.6(7) 15.4(6)(38) ^c	15.01(48)(373)
^{28}Al	3^+	β^-	2.245(2) min	1778.987	100(0) $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.00\%$	49.10(33) 49.09(34)(1288) ^{c,d}	48.22(16) 48.34(17)(1180) ^{c,d}	48.68(18)(1188)
^{28}Mg	0^+	β^-	20.915(9) h	30.6383	89.0(45) ^e $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	1.56(41) 1.56(41)(41) ^c	—	1.56(41)(41)
^{27}Mg	$1/2^+$	β^-	9.458(12) min	843.76 1014.52	71.800(20) 28.200(20) $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.11\%$	2.93(9) 2.83(20) 2.92(8)(77) ^c	2.90(11) 2.90(19) 2.91(10)(71) ^c	2.91(6)(71) ⁱ
^{27}Na	$5/2^+$	β^-	301(6) ms	984.66	87.4(6)	<0.157	<0.156	
^{26}Na	3^+	β^-	1.07128(25) s	1808.71 1128.89	99.08(50) ^f 5.88(3)	<0.115	<1.89	
^{25}Na	$5/2^+$	β^-	59.1(6) s	389.710 585.028 974.742	12.68(22) 13.00(18) 14.95(22) $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 5.4\%$	— 1.04(25) 0.98(27) 1.01(28)(27) ^c	0.652(130) ^h 1.15(18) 1.22(24) 1.18(16)(29) ^c	1.11(14)(27) ^j
^{24}Na	4^+	β^-	14.956(3) h	1368.625	99.994(2) $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	0.56(22) 0.56(22)(15) ^{c,d}	—	0.56(22)(15)
	1^+	IT	20.18(10) ms	472.2023	99.95(5) ^g $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.00\%$	0.824(45) 0.825(45)(218) ^c	0.81(6) 0.81(6)(20) ^c	0.816(35)(200) ^k
^{25}Ne	$1/2^+$	β^-	602(8) ms	89.53	95.4(8)	<0.166	<0.217	
^{24}Ne	0^+	β^-	3.38(2) min	472.2023	100.0(2)		<0.647	
^{21}Na	$3/2^+$	β^-	22.49(4) s	350.725	5.07(13) $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	2.6(10) 2.6(10)(7) ^c	—	<2.6(10)(7) ^l
^{21}F	$5/2^+$	β^-	4.158(20) s	350.725	89.6(18) ^f $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 3.32\%$	0.16(6) 0.16(6)(4) ^c	—	<0.16(6)(4) ^l

^aThe notation of the uncertainties is the same as that in Table IV.

^b I_γ uncertainty includes both $\Delta I_\gamma^{\text{rel}}$ and $\Delta I_\gamma^{\text{abs}}$ for the nuclei denoted as $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$

^cThe impurity of the enriched target (Table I) was taken into account with the decomposition process.

^dThe decay component from the mother nuclei was subtracted.

^eAbsolute I_γ was provided based on the estimation, and the uncertainty was not provided in the ENSDF. 5% uncertainty was added [27].

^f $\Delta I_\gamma^{\text{rel}}$ of these peaks are not provided in the ENSDF database and were estimated from other ΔI_γ .

^g $\Delta I_\gamma^{\text{rel}}$ is not provided in the ENSDF database. The uncertainty originating from the probability of beta decay from ^{24m}Na (0.05%) was estimated as 0.05%.

^hThe BR determined with 389.7 keV was minimal compared with the result of the other peaks obtained at J-PARC. This result was excluded from the weighted average value.

ⁱThe lower limit of ^{27}Mg is 2.75% calculated from the upper limit of ^{27}Na .

^jThe lower limit of ^{25}Na is 1.01% calculated from the upper limit of ^{25}Ne .

^kA peak at 472.2 keV originated from ^{24m}Na and ^{24}Ne . The upper limit of ^{24}Ne was estimated from the statistics during the decay measurement after stopping the beam irradiation. The lower limit of the BR of ^{24m}Na is 0.160% calculated from the upper limit of ^{24}Ne .

^lA peak at 350.7 keV could originate from ^{21}Na and ^{21}F . The background component was subtracted since the peak was also observed in the empty target measurement.

to the uncertainty of $\epsilon_{\text{stop}}^{^{29}\text{Si}}$. The determination of $\epsilon_{\text{stop}}^{^{29}\text{Si}}$ relied significantly on the absolute BR of ^{29}Al . Because ^{29}Al was produced from both ^{29}Si and ^{30}Si , $b_{^{29}\text{Al}}$ of $^{\text{nat}}\text{Si}$ was expressed as a combination of $b_{^{29}\text{Al}}^{^{29}\text{Si}}$ and $b_{^{29}\text{Al}}^{^{30}\text{Si}}$, where $b_i^j = b_i^j / \epsilon_{\text{stop}}^j$. Considering the large $b_{^{29}\text{Al}}^{^{30}\text{Si}} = 64.8(52)\%$, and smaller $b_{^{29}\text{Al}}^{^{29}\text{Si}} = 15.0(38)\%$, the uncertainty of $b_{^{29}\text{Al}}^{^{30}\text{Si}}$ strongly influenced the absolute value of $b_{^{29}\text{Al}}^{^{29}\text{Si}}$, resulting in a substantial uncertainty of approximately 25% in $\epsilon_{\text{stop}}^{^{29}\text{Si}}$.

The BRs of ^{24}Na ($T_{1/2} = 15.0$ h) and ^{28}Mg ($T_{1/2} = 20.9$ h) were not evaluated in the J-PARC experiment for all targets owing to the background component originating from the existence of silicon in the beam duct. The background component was evaluated and subtracted for short-lifetime isotopes; however, the evaluation of long-lifetime isotopes was hindered by a significant impact owing to the cumulative amount from muon irradiation prior to each measurement. In consequence, the uncertainties of the other BRs in the J-PARC

TABLE VI. Absolute production BR of each isotope produced from muonic ^{30}Si . $P_{\text{cap}} = 61.21(12)\%$ was utilized. The stopping rates were $0.349(26)\%$ and $0.261(19)\%$ in the RAL and J-PARC experiments, respectively. The same notations as those in Table II were utilized.

Nucleus	State	Decay	$T_{1/2}$	E_γ (keV)	I_γ (%) ^b	b_γ^{RAL} (%) ^a	$b_\gamma^{\text{J-PARC}}$ (%) ^a	b (%) ^a
^{30}Al	3^+	β^-	3.62(6) s	1263.13	40.1(15) ^e	14.9(6)	14.19(40)	
				1311.80	2.54(23) ^e	–	14.7(19)	
				1332.48	0.94(14) ^e	12.5(48)	–	
				2235.23	65.1(22) ^e	13.53(49)	13.84(49)	
				2595.39	5.77(22) ^e	–	13.4(10)	
				3498.33	32.6(14) ^e	–	13.9(7)	
^{29}Al	$5/2^+$	β^-	6.56(6) min		$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	14.12(44)(115) ^c	14.04(36)(113) ^c	14.08(32)(113)
				1152.57	1.031(27)	68.(8)	58.2(32)	
				1273.36	91.3(9) ^f	65.8(7)	64.23(48)	
				2028.09	3.514(27)	62.5(19)	68.8(17)	
				2425.73	5.23(6)	68.3(16)	67.3(15)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.07\%$	65.4(9)(53) ^{c,d}	64.3(7)(52) ^{c,d}	64.8(7)(52)
^{28}Al	3^+	β^-	2.245(2) min	1778.987	100(0)	15.39(20)	14.13(10)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.0\%$	13.94(22)(114) ^{c,d}	13.94(10)(113) ^{c,d}	13.94(10)(112)
^{29}Mg	$3/2^+$	β^-	1.30(12) s	1398.0	16.4(11)	–	0.54(22)	
				1754.2	9.90(72)	–	0.82(26)	
^{28}Mg	0^+	β^-	20.915(9) h		$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 13.89\%$		0.66(24)(5) ^c	0.66(24)(5)
				30.6383	89.0(45) ^g	1.99(41)	–	
				400.6	35.9(18) ^g	1.97(15)	–	
				941.7	36.3(18) ^g	1.79(16)	–	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	1.90(11)(15) ^c		1.90(11)(15) ^k
^{27}Mg	$1/2^+$	β^-	9.458(12) min	843.76	71.800(20)	1.59(6)	1.70(12)	
				1014.52	28.200(20)	1.91(18)	1.51(15)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.11\%$	1.61(6)(13) ^c	1.63(9)(13) ^c	1.62(5)(13) ^l
^{28}Na	1^+	β^-	30.5(4) ms	1473.5	37(5)	<0.43	<0.29	
^{27}Na	$5/2^+$	β^-	301(6) ms	984.66	87.4(6)	<0.12	<0.10	
^{26}Na	3^+	β^-	1.07128(25) s	1808.71	99.08(50) ^f	0.290(44)	–	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.03\%$	0.291(44)(24) ^c		0.291(44)(24) ^m
^{25}Na	$5/2^+$	β^-	59.1(6) s	389.710	12.68(22)	1.33(46)	0.60(18) ^j	
				585.028	13.00(18)	1.4(6)	1.04(19)	
				974.742	14.95(22)	–	1.05(25)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 5.4\%$	1.36(37)(11) ^c	1.04(16)(8) ^c	1.10(15)(9) ⁿ
^{24}Na	4^+	β^-	14.956(3) h	1368.625	99.994(2)	0.21(7)	–	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$	0.21(7)(2) ^{c,d}		0.21(7)(2)
	1^+	IT	20.18(10) ms	472.2023	99.95(5) ^h	0.520(37)	0.541(36)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.0\%$	0.517(37)(43) ^c	0.539(36)(44) ^c	<0.528(26)(43) ^o
^{25}Ne	$1/2^+$	β^-	602(8) ms	89.53	95.4(8)	<0.33	<0.26	
^{24}Ne	0^+	β^-	3.38(2) min	472.2023	100.0(2)	0.513(35)	0.518(34)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = 0.0\%$	0.513(35)(42) ^c	0.517(35)(42) ^c	<0.515(25)(41) ^o

TABLE VI. (Continued.)

Nucleus	State	Decay	$T_{1/2}$	E_γ (keV)	I_γ (%) ^b	b_γ^{RAL} (%) ^a	$b_\gamma^{\text{J-PARC}}$ (%) ^a	b (%) ^a
²² F	(4 ⁺)	β^-	4.230(40) s	2082.6	81.9(20)	<0.113	<0.104	
²⁰ Na	2 ⁺	ϵ	447.9(23) ms	1633.602	79.3(11)	–	0.13(6)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$		0.13(6)(1) ^c	<0.13(6)(1) ^p
²⁰ F	2 ⁺	β^-	11.163(8) s	1633.602	99.9995(10) ⁱ	–	0.105(45)	
					$\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$		0.105(45)(8) ^c	<0.105(45)(8) ^p

^aThe notation of the uncertainties is the same as those in Table IV.

^b I_γ uncertainty includes both $\Delta I_\gamma^{\text{rel}}$ and $\Delta I_\gamma^{\text{abs}}$ for the nuclei denoted as $\Delta I_\gamma^{\text{abs}}/I_\gamma^{\text{abs}} = \text{incl.}$

^cThe impurity of the enriched target (Table I) was taken into account with the decomposition process.

^dThe decay component from the mother nuclei was subtracted.

^eAbsolute I_γ was not provided in the ENSDF and was calculated from the intensity of β decay to each energy level and the probability of γ -ray transition from each state [29–31]. The uncertainties contained absolute uncertainty.

^f $\Delta I_\gamma^{\text{rel}}$ of these peaks are not provided in the ENSDF database and were estimated from other ΔI_γ .

^gAbsolute I_γ was provided based on the estimation. The uncertainty was not provided in the ENSDF. 5% uncertainty was added [27].

^h $\Delta I_\gamma^{\text{rel}}$ is not provided in the ENSDF database. The uncertainty originating from the probability of beta decay from ^{24m}Na (0.05%) was estimated as 0.05%.

ⁱ $\Delta I_\gamma^{\text{rel}}$ of this peak is not given in the ENSDF database and was estimated from the probability of the beta decay to the ground state.

^jThe BR determined with 389.7 keV was minimal compared with the result of the other peaks obtained at J-PARC. This result was excluded from the weighted average value.

^kThe lower limit of ²⁸Mg is 1.65% calculated from the upper limit of ²⁸Na.

^lThe lower limit of ²⁷Mg is 1.53% calculated from the upper limit of ²⁷Na.

^mThe lower limit of ²⁶Na could not be restricted from the upper limit of ²⁶Ne. However, the 1n3p channel can be assumed as negligible inferring from the other results.

ⁿThe lower limit of ²⁵Na is 1.10% calculated from the upper limit of ²⁵Ne.

^oA peak at 472.2 keV originated from ^{24m}Na and ²⁴Ne. The upper limit of these isotopes could not be restricted by the other energy peaks, and the production BR of these isotopes both represent the upper limit.

^pA peak at 1633.6 keV could originate from ²⁰Na and ²⁰F. The upper limit of these isotopes could not be restricted by the other energy peaks, and the production BR of these isotopes both represent the upper limit.

experiment increased owing to the uncertainty resulting from the background subtraction process.

In ²⁴Na, an isomeric state with a lifetime of 20 ms was observed, decaying through the isomeric transition to the ground state by emitting the γ ray with the energy of 472.2 keV. The population of the isomeric state at 472.2 keV is not only a result of direct production following the particle emission from muon nuclear capture but also a result of the β decay of ²⁴Ne. By leveraging the decay of ²⁴Ne into ²⁴Na through the emission of γ rays at 472.2 and 874.4 keV, the upper limit of the BR of ²⁴Ne was estimated using the upper count limit at 874.4 keV. The peak count of 472.2 keV after the beam irradiation was also utilized to the estimation of the upper limit of ²⁴Ne, considering 20.2 ms lifetime of ^{24m}Na, which implies that its decay γ ray should not be detected in decay measurement after the beam irradiation. The lower limit of ^{24m}Na was inferred based on the upper limit of ²⁴Ne except for the ³⁰Si target. For the ³⁰Si target, only the upper limits of the BR of ^{24m}Na and ²⁴Ne were determined. The BR of ²⁴Na (ground state) was inferred by considering the creation from the ^{24m}Na, assuming that all the yield of 472.2-keV peaks were produced by ^{24m}Na. The longer lifetime of ²⁴Na compared to both ^{24m}Na and ²⁴Ne, along with minimal differences in the upper limits of their BRs, ensured that the breakdown of these isotopes did not impact the determination of the ground state BR of ²⁴Na.

The production BR of ²⁵Na determined by the yield of a 389.7-keV peak demonstrated a notably lower value when compared with results obtained from other energy peaks at 585.0 and 974.7 keV. This discrepancy was observed solely in the analysis of silicon targets in the J-PARC experiment, as the RAL experiment and the analysis of the aluminum target yielded a result (389.7 keV) consistent with the estimation from other energy peaks. The efficiency of the germanium detector does not strongly affect the yield of 389.7-keV, even considering the effect of self-absorption. Background energy peaks at these energies were absent, and precautions were taken to avoid the potential contamination from single or double escape of higher energy γ rays. The other possibilities of the contamination, such as overlap with the γ -ray emission from ²⁵Al and ²²Mg, were considered, but were insufficient to explain the discrepancy. The cause of the significantly lower yield of 389.7 keV peak remains uncertain, leading to the exclusion of this data when determining the BR of ²⁵Na.

The transition γ ray from ²⁰Ne at an energy level of 1633.6 keV was detected with all the targets except for ²⁹Si, with the transition γ ray from ²¹Ne at 350.7 keV also observed in the ²⁷Al and ²⁹Si targets. The detected γ rays of ²⁰Ne and ²¹Ne could originate from either ²⁰F and ²⁰Na or ²¹F and ²¹Na, respectively. The origin could not be determined based on available information, such as lifetime and the upper limit of the other nondetected γ rays. Consequently, the BRs of ^{20,21}F

and $^{20,21}\text{Na}$ were listed as upper limits. In the ^{27}Al target, the presence of ^{23}Ne and ^{23}Mg was also presented as an upper limit owing to the possibility that the detected transition γ ray from $5/2^+$ to $3/2^+$ state in ^{23}Na could originate from either isotope.

Notably, the production BR may be influenced by electrons and neutrons present in the beam or the surroundings. Electrons within the muon beam or generated during the muon decay process ($\mu^- \rightarrow \nu_\mu + \bar{\nu}_e + e^-$) can transmute target material through (γ , $xnyp$) reactions, facilitated by photons from bremsstrahlung radiation. Environmental neutrons, as well as neutrons produced by muon nuclear capture reactions, can also react with the target material. The potential for the background component to be generated by neutrons and electrons was assessed through a Monte Carlo simulation code [32] that will be explained in Sec. VI. The impact of electrons was deemed negligible, assuming the number of electrons was roughly twice as large as the stopping muon in the target for decay electrons and 30 times larger for beam-contaminated electrons. Environmental neutrons were estimated as thermal neutrons, which did not influence the production BR owing to the prevalence of (n , γ) or (n , n). However, the neutron emitted from the muon nuclear capture reaction could potentially impact the production BR results. The maximum potential contamination resulting from neutron irradiation was estimated to be approximately 3% for the $0n0p$ channel of $\mu^- + ^{27}\text{Al}$ and $\mu^- + ^{28}\text{Si}$, assuming that the number of muons stopping in the surrounding material was equivalent to that of the target. This contamination level was comparable to the uncertainty of the present measurement and was not corrected in the result. Any potential contamination from these neutrons in channels other than the $0n0p$ channel was smaller than the uncertainty observed.

VI. DISCUSSION

In this section, the present results were compared with previous measurements and discussed in terms of isotope dependence and the even-odd effect of neutron and proton numbers. Additionally, the results were compared with theoretical model calculations; muon interaction models implemented in a Monte Carlo simulation using the particle and heavy ions transport code system ver. 3.33 [32] and the microscopic and evaporation model [6] referred to as PHITS and microscopic and evaporation model (MEM) calculation hereinafter. The excitation energy distribution resulting from the muon nuclear capture reaction was estimated using the present results and model calculations.

A. Comparison with previous measurements

The production BR results of ^{27}Al and $^{\text{nat},28}\text{Si}$ were compared with those of previous studies, utilizing the activation method [33–38] and prompt γ -ray measurements [38–42], as listed in Table VII. Notably, the muon nuclear capture probabilities (P_{cap}) utilized in the analysis of the previous studies differed from those in the present study [7] by approximately 1%. The discrepancy was not corrected for the comparison owing to the lack of description of values in some research,

TABLE VII. Comparison of the BR (in percentage) obtained in the present study with that from previous measurements. The absolute production BR obtained with the present study, previous activation measurement, and the lower limit of BR obtained from the prompt γ -ray measurement were presented for each reaction channel.

Reaction	Present	Previous	Methods
$^{27}\text{Al}(\mu^-, \nu_\mu)^{27}\text{Mg}$	9.90(33)	12.0(7) >9.7 (11) >6.8 (7)	Activation [37] ^b Prompt γ [42] Prompt γ [40]
$^{27}\text{Al}(\mu^-, \nu_\mu np)^{25}\text{Na}$	2.52(18)	2.8(4)	Activation [36]
$^{27}\text{Al}(\mu^-, \nu_\mu 2np)^{24}\text{Na}$	1.61(19)	2.1(2) 3.5(8)	Activation [37] ^b Activation [35]
$^{27}\text{Al}(\mu^-, \nu_\mu 2n2p)^{23}\text{Ne}$	0.80(6)	0.76(11)	Activation [36]
$^{\text{nat}}\text{Si}(\mu^-, \nu_\mu x)^{28}\text{Al}$	20.1(7)	22.8(25) 31(3) >4.2 (6)	Activation [37] ^b Activation [33] Prompt γ [40]
$^{\text{nat}}\text{Si}(\mu^-, \nu_\mu x)^{27}\text{Mg}$	2.94(15)	5.6(8)	Activation [34]
$^{\text{nat}}\text{Si}(\mu^-, \nu_\mu x)^{24}\text{Na}$	1.67(12)	3.4(2)	Activation [37] ^b
$^{28}\text{Si}(\mu^-, \nu_\mu)^{28}\text{Al}$	18.9(9)	26(3) >16.6 (12) ^a >16.9 (15) >20.3	Activation [38] ^b Prompt γ [42] Prompt γ [41] ^b Prompt γ [38] ^b
$^{28}\text{Si}(\mu^-, \nu_\mu p)^{27}\text{Mg}$	2.87(14)	>2.5 (4) ^a >1.9 (2)	Prompt γ [42] Prompt γ [38] ^b
$^{28}\text{Si}(\mu^-, \nu_\mu 2n2p)^{24}\text{Na}$	1.71(13)	>0.5 (5) ^a	Prompt γ [42]

^aThese values were obtained using natural abundance silicon and estimated to the BR of ^{28}Si in the paper [42].

^bThese measurements used oxide target, Al_2O_3 , SiO_2 , or $^{28}\text{SiO}_2$.

and the magnitude of the effect was smaller than the uncertainty of BRs.

Previous activation measurements were conducted for ^{27}Al , ^{28}Si , and $^{\text{nat}}\text{Si}$. Wytttenbach *et al.* [36] measured the activation of ^{27}Al targets, with the results of the production BRs of ^{25}Na and ^{23}Ne being comparable with the present results. Miller *et al.* measured enriched $^{28}\text{SiO}_2$, whereas Heisinger *et al.* measured natural abundance Al_2O_3 and SiO_2 targets [37,38], both showing systematically larger BRs. The discrepancy between the present results and those of Miller *et al.* and Heisinger *et al.* may be attributed to the uncertainty of the stopping number of incoming muons. Muonic x rays were employed to measure the muon attachment probability for each isotope, as the measurements were performed with oxide targets, potentially underestimating the stopping number of muons. Discrepancies with the other previous studies [33–35] could result from the measurement method of the stopping number of muons and the inferior energy resolution of the detector. Additionally, previous activation measurements did not consider the effect of production from mother nuclei, which are also generated by the muon nuclear capture reaction, such as the β^- decay component from the ^{28}Mg in the BR of ^{28}Al .

The prompt γ -ray measurement technique determines BR by analyzing the γ -ray emissions that occur during the deexcitation of nuclei following particle emission. By identifying the

reaction products based on the characteristic γ -ray energies, the BRs can be calculated by summing the total transition yield to the ground state. It is important to note that prompt γ -ray measurements provide the lower limit of BR, as there may be missing yields with undefined or weak transitions, and the direct population to the ground state. Previous studies utilizing prompt γ -ray measurements have yielded results that are consistent with current findings, except for the study conducted by Miller *et al.* The overestimation of the results presented by Miller *et al.* can be attributed to the incorrect assignment of certain γ -ray energies of ^{28}Al , as highlighted by Measday *et al.* [42]. Upon comparing the results of our current activation measurement with the previous prompt γ -ray measurement [42] for the $0n0p$ channel, the prompt γ -ray measurement covered nearly all [98(12)%] the BR of $0n0p$ channel in $\mu^- + ^{27}\text{Al}$, and 88(8)% of that of the ^{28}Si target. This indicates minimal missing yields and direct ground-state populations in the $0n0p$ channel, contrary to the larger estimates of the missing yield in absolute BR proposed by Measday *et al.* of 3% for ^{27}Al and 9.4% for ^{28}Si [42], which were based on the comparison of their results with previous activation measurements as can be seen in Table VII.

The direct measurement of the emitted charged particles [3,4,43–46], neutron [47,48], and neutron multiplicity measurement [49] have been previously reported. However, direct comparison with most charged-particle measurements is challenging because they usually focus on high-energy particle emission, whereas most components of the emitted particles typically have low energy. For example, the charged-particle emission probability above 1.5 MeV was reported as 15(2)% [43] for a natural silicon target without particle identification. The measured BR of the charged particles was 5.2(2)%, which is the sum of the charged-particle emission channels in Table III. The discrepancy in the BR is primarily attributed to the production of ^{26}Mg , with a BR exceeding 8.4(8)% [42], which could not be measured in the present experiment. The distribution of neutron multiplicity was measured [49] and evaluated [2] as $0n$ [9(6)%], $1n$ [75(10)%], $2n$ [5(10)%], and $3n$ [9(6)%] for ^{27}Al , and $0n$ [36(6)%], $1n$ [49(10)%], $2n$ [14(6)%] and $3n$ [1(1)%] for a natural silicon target, assuming that higher neutron emissions exceeding three did not occur. The no-neutron emission channel includes the $0n0p$ channel as well as charged-particle emission channels that do not involve neutron emission, such as deuteron or α emission. The present result of no neutron emission probabilities were 11.7(4)% and >23.7% for ^{27}Al and ^{28}Si , respectively. These probabilities were derived from a combination of channels, including $0n0p$, $0n1p$, a portion of $1n1p$, $1n2p$, and $2n2p$ channels, assuming that the $xn2p$ channels represent helium isotope emission. The ratio of one neutron and one proton emission to deuteron emission in $1n1p$ channel was estimated from the number of proton and deuteron emissions from the direct particle measurement [3] to be 1:5, consistent with the estimation in Ref. [36]. Despite the significant uncertainty in multiplicity, this ratio remains consistent with previous research. The several neutron emission probabilities were difficult to compare owing to the complexity of neutron emission accompanied by charged-particle emission and the substantial uncertainty in multiplicities.

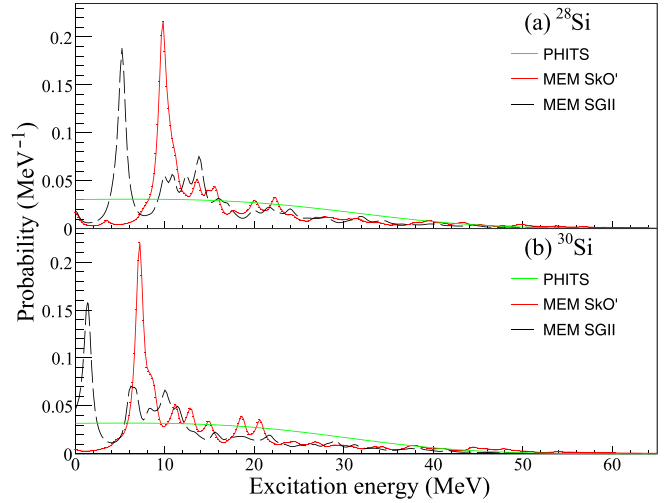


FIG. 6. Excitation function of the muon nuclear capture reaction of (a) ^{28}Si and (b) ^{30}Si , calculated using the Singer model [50] used in PHITS (green solid line), as well as the MEM [6] with the effective interaction of SkO' (red solid line) and SGII (black dotted line).

Overall, the present results were comparable with the previous findings from prompt γ -ray measurements and direct particle detection. This measurement represents the first reliable absolute data of the production BR for muon nuclear capture of ^{27}Al and silicon isotopes by utilizing the advantage of the in-beam activation method.

B. Trend of the results

In this section, the structure of the excitation function and the particle emission mechanism following the muon nuclear capture reaction is discussed.

The probability of particle emission after muon nuclear capture depends on the particle emission processes, and the excitation energy of the nucleus, and the threshold energy required for each emissions. Three particle emission processes from the muon nuclear capture reaction have been proposed: direct, preequilibrium, and evaporation processes. Notably, particles emitted through the direct and preequilibrium processes carry higher energy compared with those produced by the evaporation process. The excitation function, which represents the energy distribution of the excitation, has been discussed phenomenologically [50–58] and using some microscopic calculations [6,59,60]. The excitation functions calculated using the Singer model [50], as well as the microscopic and evaporation model [6] with the effective interaction of SkO' and SGII, are shown in Fig. 6; details will be discussed in Sec. VI C. The excitation function has an energy peak at approximately 5–20 MeV, extending up to 100 MeV. However, the detailed structure of the excitation function remains elusive as direct measurement is hindered by the difficulty associated with detecting emitted muon neutrinos. The probability of particle emission is also influenced by the threshold energy required for each particle emission, particularly for the evaporation process. Figure 7 shows the threshold energy below 40 MeV calculated from the atomic

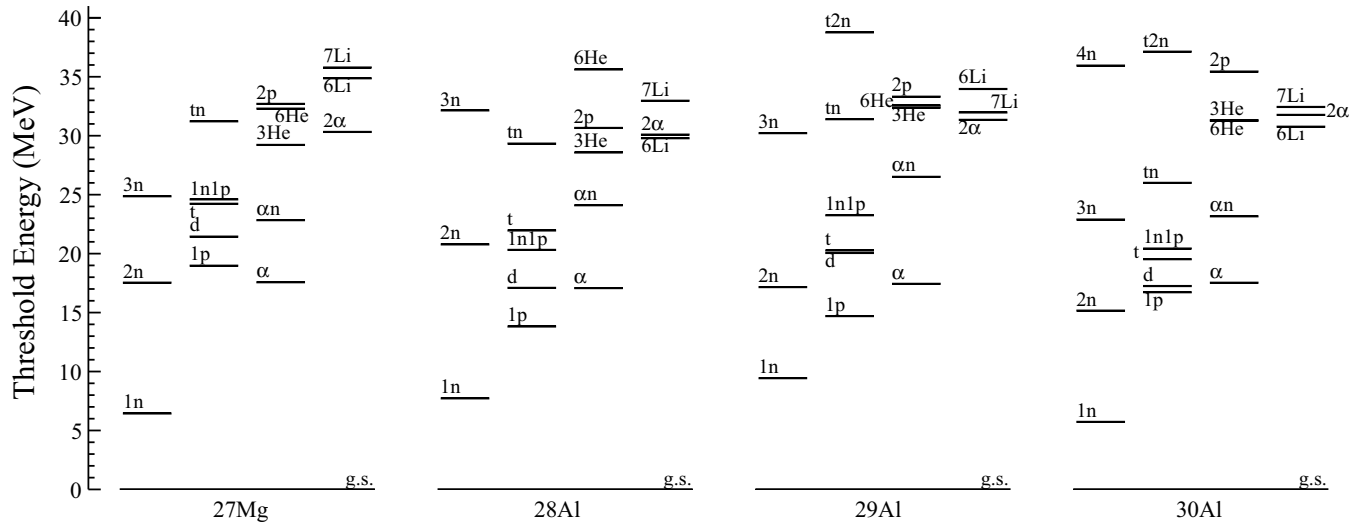


FIG. 7. Threshold energy of each particle emission for ^{27}Mg and $^{28,29,30}\text{Al}$, representing reaction products of muon nuclear capture on ^{27}Al and $^{28,29,30}\text{Si}$. For the charged-particle emission, the contribution of the Coulomb barrier was added to the threshold energy, utilizing the classical model (see text). The thresholds of both one neutron and one proton emission, as well as deuteron emission, are presented for $1n1p$ emission. The threshold energy for other multiple particle emissions was calculated assuming that they would be emitted as light complex particles, such as helium isotopes, for the $xn2p$ emission channels.

mass table (AME2020) [61] for ^{27}Mg and $^{28,29,30}\text{Al}$. These isotopes were initially formed through muon nuclear capture of ^{27}Al and $^{28,29,30}\text{Si}$. For the charged-particle emission, the Coulomb barrier is added to the threshold energy in the figure. The Coulomb barrier was calculated using Eq. (6) of Ref. [36],

$$V_c = \frac{e^2}{4\pi\epsilon_0} \frac{zZ}{r_0 A^{1/3} + \rho_0}, \quad (7)$$

which expresses the systematic Coulomb barrier with mass and atomic numbers. The threshold energy for the emission of multiple particles was estimated based on the assumption that they are emitted as light particles, such as helium isotopes for $2p$ emission channels.

The results of the absolute BR obtained through the present measurement are listed with the absolute uncertainty in Tables VIII–XI. In the case where channels could not be distinguished based on detected β -delayed γ -ray energy, for example, such as the $4n0p$ channel (^{23}Mg) and $2n2p$ channel (^{23}Na) of ^{27}Al , the higher-threshold-energy channel was excluded. Previous values for the long-life isotopes and stable nuclei that were not measured in this experiment are also included. The lower limits from the prompt γ -ray measurement and reliable values from the activation measurement, which were consistent with the present study, were selected as the previous results. The sum of experimentally measured BRs for each isotope was also provided. Tables VIII–XI reveal that neutron emission without charged-particle emission is predominant for all target isotopes, with the $1n0p$ channel demonstrating the maximum BR. By considering the threshold energy of each particle emission and the present result, the average excitation function was estimated at approximately 10–20 MeV, with the excitation energy exceeding 30 MeV being observed. The discussion below focuses on the variance in

particle emission probability between aluminum and various silicon isotopes.

1. Even-odd isotope dependence of no particle emission channel

Comparing the neutron emission probabilities of ^{27}Al and $^{28,29,30}\text{Si}$ in Table VIII, the $0n0p$ channel of $\mu^- + ^{27}\text{Al}$, 9.9(3)%, is obviously smaller than that of silicon isotopes. Although the $1n0p$ channel of ^{27}Al and ^{28}Si were not measured in the current measurement owing to their stable residuals, the $1n0p$ channel appeared to be larger in aluminum compared with silicon isotopes, drawn from the lower limit observed in previous measurements and estimation based on measured charged-particle emission probabilities. The smaller $0n0p$ channel in $\mu^- + ^{27}\text{Al}$ cannot be explained by the difference of separation energy between ^{27}Mg and $^{28,29,30}\text{Al}$. The mass differences of the initial and final nuclei of the muon nuclear capture reaction were 2.6, 4.6, 3.7, and 8.6 MeV for ^{27}Al and $^{28,29,30}\text{Si}$. Although the mass difference of the reaction of ^{27}Al was lower than those of the other silicon isotopes, it did not directly influence the average excitation energy. For example, in the Singer model [50] that describes a phenomenological excitation function, the amount of the mass difference influences the available energy for the reaction, with the difference in mean excitation energy estimated to be less than 1 MeV. This difference does not result in a twofold change in the BR of the $0n0p$ channel. Furthermore, the even-odd and shell effects do not impact the direct and preequilibrium emission processes [62] when assuming that the excitation energy remains constant across isotopes. Therefore, the discrepancy observed in the $0n0p$ channel cannot be explained by the characteristics of particle emission processes but rather may result from the difference in the excitation function. As the average excitation energy does not differ drastically with the mass and atomic number [53], the difference could lie in the

TABLE VIII. Summary of the production BR (in percentage): neutron emission.

Target	0n0p	1n0p	2n0p	3n0p	Sum
²⁷ Al	9.90(33)	>70(6) ^b	>7(1) ^c	>2 ^c	>89(6)
²⁸ Si	18.9(9)	>28(2) ^a	>2.8(5) ^a		>49.7(23)
²⁹ Si	15.0(38)	49(12)	–	–	64(16)
³⁰ Si	14.1(12)	65(5)	13.9(11)	–	93(7)

^aThese values were obtained with natural silicon and modified to the BR of ²⁸Si with some assumptions in the reference [42].

^bSourced from Ref. [40]

^cSourced from Ref. [42]

strength of the transition peak below the threshold energy, influenced by the shell structure.

The increased neutron emission probabilities in odd-*Z* nuclei have been consistent with previous measurements, particularly in medium-mass nuclei. A comprehensive overview of the BRs of the 0n0p channel, focusing on the even-odd nature of atomic numbers, is shown in Fig. 8. The BRs obtained using the activation method were indicated using closed square and circle symbols for even-*Z* and odd-*Z* nuclei, respectively. The data obtained using prompt γ -ray measurement are represented with open symbols, assuming that the prompt γ -ray measurements covered most BR in the 0n0p channel, as discussed in Sec. VI A. Looking at Fig. 8, the BR of the 0n0p channel in muon nuclear capture of odd-*Z* nuclei has a smaller BR of 0n0p, whereas even-*Z* nuclei demonstrate a larger BR. This trend is consistent across medium-mass even-*Z*-even-*N* and odd-*Z*-even-*N* nuclei in light and medium mass nuclei.

In the muon nuclear capture reaction, a strong transition to the 1⁺ state has been observed, as evidenced by the excited levels obtained through prompt γ -ray measurements [2,38,67]. This knowledge suggests that the Gamow-Teller (GT) transition plays a crucial role in muon capture, analogously to its importance in pion capture and β^+ decay. Thus, the even-odd *Z* dependence observed in the 0n0p channel may be attributed to the even-odd effect in the GT transition, induced by the proton-neutron (*pn*) pair creation and known to influence the GT strength in the low-energy regions [68–71]. When the configuration of the GT state comprises a *pn* pair on a closed-shell nucleus with particle-particle properties, specifically when the *pn* pair is separated from the core and demonstrates deuteron-like behavior in the final

TABLE IX. Summary of the production BR (in percentage): one proton (including deuteron and triton) emission.

Target	0n1p	1n1p	2n1p	3n1p	4n1p	Sum
²⁷ Al	0.78(6)	2.52(18)	1.61(19)	–	0.05(1) ^b	4.97(30)
²⁸ Si	2.87(14)	>8.4 (8) ^a	>1.5 (1) ^a	–		>12.8 (8)
²⁹ Si	1.6(6)	2.9(7)	–	–	–	4.5(12)
³⁰ Si	0.66(25)	1.90(19)	1.62(14)	–	–	4.17(43)

^aThese values were obtained with natural silicon and modified to the BR of ²⁸Si with some assumptions in Ref. [42].

^bSourced from Ref. [37]

TABLE X. Summary of the production BR (in percentage): two protons (helium isotopes) emission.

Target	0n2p	1n2p	2n2p	3n2p	4n2p	Sum
²⁷ Al		0.038(7) ^d	0.80(6) ^e	–	–	0.84(6)
²⁸ Si		0.24(17)	1.71(13)	>0.5 (5) ^a	0.15(3) ^b	>2.6 (5)
²⁹ Si			1.11(31)	1.39(42)	–	2.5(7)
³⁰ Si			0.29(5)	1.10(18)	0.73(8) ^c	2.11(24)

^aThis value was obtained with natural silicon and modified to the BR of ²⁸Si with some assumptions in Ref. [42].

^bThis value resulted from natural silicon [37]; however, 92% of the components of natural silicon are ²⁸Si, which was not corrected.

^cA BR of ²⁴Ne was deemed negligible.

^dSourced from Ref. [36]

^eA BR of ²³Mg was deemed negligible.

states, a strong low-energy peak appears as a result of strong *T* = 0 pairing correlations. This phenomenon has thus far been investigated through the examination of GT strength in closed-shell and two nucleons with *N* = *Z* nuclei [70,72,73]. Additionally, a similar effect owing to the *pn* pair in spin-dipole excitation has been investigated [74]. In the transition from ²⁸Si to ²⁸Al with the muon nuclear capture reaction, the primary shell configuration is the initial (0⁺, *T* = 1) state of two protons in *d*_{5/2} to the final (1⁺, *T* = 0) state of one proton in *d*_{5/2} and one neutron in either *d*_{5/2} or *d*_{3/2}, in which the *pn* pairing lower the energy of GT strength peak and enhance its intensity. Conversely, the transition from ²⁷Al to ²⁷Mg remains unaffected by the *pn* pair owing to the even-*Z* of the final state.

To discuss the relationship between the enhancement of GT strength at low energy and the even-odd *Z* dependence of the 0n0p channel, the GT strength, *B*(GT), of ²⁷Al and ²⁸Si was calculated using the KSHELL code [75] with USD-A interaction. The calculated *B*(GT) values, along with the one neutron separation, energy are shown in Fig. 9. The cumulative GT strength below the one-neutron separation energy of the final state post muon nuclear capture reaction amounts to 1.77 and 3.36 for ²⁷Al and ²⁸Si, respectively. These ratios are comparable with those of the BR of 0n0p channels of 9.9(3)% and 18.9(9)% for ²⁷Al and ²⁸Si, respectively. Therefore, the GT transition strength at low energy is enhanced by the *pn* pairing effect and influences the peak structure of the excitation function resulting from muon nuclear capture. This variation in the peak structure at low energy leads to the even-odd effect observed in particle emission probabilities. The

TABLE XI. Summary of the production BR (in percentage): other charged-particle emissions.

Target	3n3p	4n3p	4n4p	6n4p
²⁷ Al	0.096(37) ^b	0.102(21) ^a		–
²⁸ Si	–	–	0.116(21) ^a	
²⁹ Si		–	0.16(7) ^b	–
³⁰ Si			–	0.105(46) ^a

^aA BR of ²⁰Na was deemed negligible.

^bA BR of ²¹Na was deemed negligible.

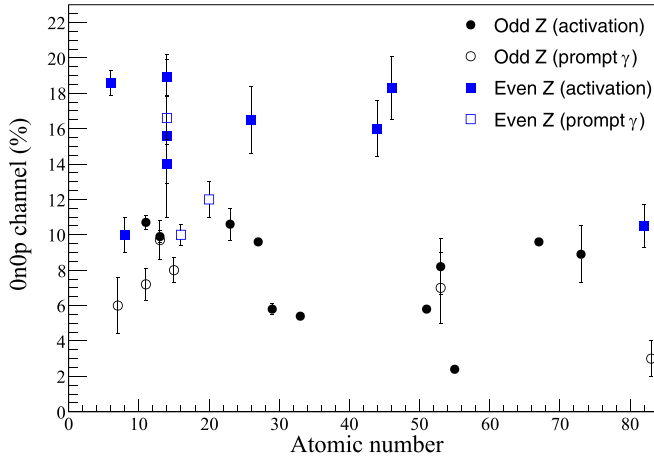


FIG. 8. Summary of the BR of the $0n0p$ channel obtained previously together with the present result. The values are summarized focusing on the even-odd distribution of the proton number. Previous results of the activation method are indicated using closed square and circle symbols for even- Z and odd- Z nuclei, respectively. The data obtained using prompt γ -ray measurement are represented with open symbols.

The present results of ^{27}Al and $^{28,29,30}\text{Si}$ were utilized in activation results, and the results of prompt γ -ray measurement of ^{27}Al and ^{nat}Si were sourced from Ref. [42]. The activation data for the isotopes ^{23}Na , ^{51}V , ^{56}Fe , ^{59}Co , ^{65}Cu , ^{75}As , ^{104}Ru , ^{123}Sb , ^{133}Cs , ^{165}Ho , ^{181}Ta , and ^{208}Pb were obtained through activation measurements conducted by Wyttenbach *et al.*, as referenced in note 9 added in proof of Ref. [57]. The results for ^{56}Fe , ^{65}Cu , ^{123}Sb , and ^{208}Pb were likely obtained using natural abundance targets and corrected to the value of enriched isotopes owing to their presence as the heaviest isotopes in natural abundance. The isotopes ^{12}C and ^{16}O were evaluated by Measday [2]. ^{14}N was sourced from Ref. [63] (prompt γ); the prompt γ -ray results of ^{23}Na , ^{31}P , and ^{32}S were sourced from Ref. [41], ^{40}Ca was sourced from Ref. [64] (prompt γ), ^{104}Pd was sourced from Ref. [1] (activation), ^{127}I was sourced from Ref. [65] for activation and Ref. [66] for prompt γ measurement, and ^{209}Bi was sourced from Ref. [66] (prompt γ). The natural abundance targets with high enrichment were not adjusted for the BR of enriched isotopes, including $^6\text{C}_6$:98.9%, $^7\text{N}_7$:99.6%, $^8\text{O}_8$:99.8%, $^{16}\text{S}_{16}$:95.0%, $^{20}\text{Ca}_{20}$:96.9%, and $^{23}\text{V}_{28}$:99.8%.

enhancement in $B(\text{GT})$ does not significantly impact the excitation function in the high-energy region, which is consistent with the experimental findings indicating minimal differences in multiple charged-particle emissions or preequilibrium components [3,4]. Heavy nuclei are unaffected by the pn pair owing to an excess of neutrons, leading to a particle-hole-type excitation. This observation is further consistent with the small BR of lead, reported at 10.5(12)% [57], and the previous result of the temperature of compound nuclei [76], suggesting a higher neutron emission probability for lead compared with bismuth. The limited increase in the $0n0p$ in closed-shell nuclei, such as ^{32}S ($0n0p$: 9.9(6)% [41]) and ^{40}Ca ($0n0p$: 12(1)% [64]), can be attributed to the filled shells, making it challenging to generate pairs through GT transition. Notably, the impact of the enhanced low-energy transition strength

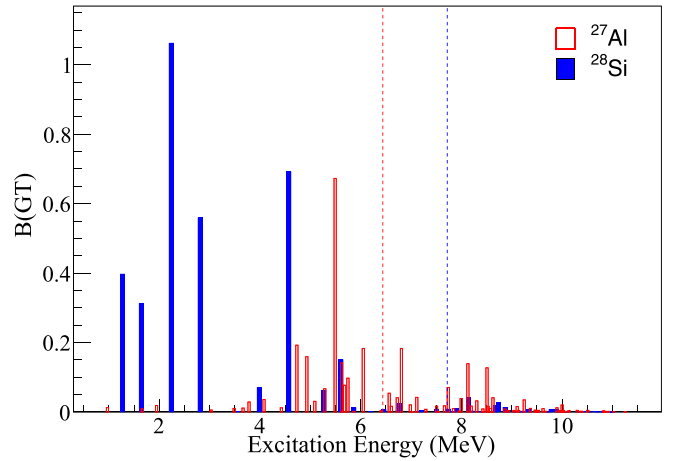


FIG. 9. Gamow-Teller strength of the transition from ^{27}Al to ^{27}Mg (red) and from ^{28}Si to ^{28}Al (blue) calculated utilizing KSHELL. The one-neutron separation energies of ^{27}Mg (6.44 MeV) and ^{28}Al (7.72 MeV) are also represented using vertical dotted lines.

owing to the pn pair on the $0n0p$ channel may involve not only GT but also spin-dipole resonance components. Furthermore, to eliminate the possibility of atomic number dependence on preequilibrium emission and significant shifts in excitation energy, direct energy distribution measurements of emitted particles are required.

In $^{28,29,30}\text{Si}$, the BRs of the $0n0p$ channel decreases along the neutron number, as shown in Table VIII. The discrepancy in the $0n0p$ channel between ^{28}Si and ^{30}Si can be understood by the neutron separation energy. When considering the mean excitation energy ranging from 10 to 20 MeV and assuming it does not differ much in ^{28}Si and ^{30}Si , a higher neutron separation energy results in a higher $0n0p$ channel for ^{28}Si . However, the BR of $0n0p$ channels of $\mu^- + ^{28}\text{Si}$ is larger than that of ^{29}Si , although the neutron separation energy of ^{28}Al is smaller than that of ^{29}Al . The direct and preequilibrium emissions have been known not to be influenced by the even-odd and shell effects [62]. The mass differences of the initial and final nuclei of the muon nuclear capture reaction may influence the average excitation energy up to 1 MeV and may reduce the BR of the $0n0p$ channel in $\mu^- + ^{29}\text{Si}$. The difference in the detailed structure of the excitation function below the threshold energy, as discussed, may also affect the lower $0n0p$ channel of ^{29}Si than ^{28}Si . The impact of strong GT strength on the low-energy region resulting from the pn pair is believed to be decreased for ^{29}Si owing to the even neutron number in the final state, leading to a decrease in BR of the $0n0p$ channel. These cumulative effects may lead to an increased neutron emission from the compound nuclei of odd- N nuclei.

2. Charged-particle emission

The BRs of one-proton and two-proton emission channels are listed in Tables IX and X. When comparing the BRs of charged-particle emission from isotopically enriched

silicon targets, the BRs of the $0n1p$ and $2n2p$ channels decrease with increasing neutron excess. This trend indicates that the probability of charged-particle emission without neutron emission depends on the neutron excess, assuming most of the $2n2p$ channel corresponds to α particle emission. Focusing on the $xn2p$ ($x > 2$) channels of ^{28}Si and ^{30}Si , the neutron emission accompanied by charged-particle emissions becomes more prominent with increasing neutron excess. The presence of large $3n2p$ and $4n2p$ channels in ^{30}Si compared with the $2n2p$ channel cannot be explained by the separation energy of these channels, considering the decrease in excitation energy distribution above approximately 10–20 MeV. The total charged-particle emission probabilities also appear to be influenced by the neutron excess, with a decrease in total charged-particle emission as neutron excess increases. This trend is generally attributed to differences in threshold energy. Still, the probabilities of charged-particle emission are affected by the Coulomb barrier, competing channels having similar threshold energies, and emission from direct and preequilibrium processes as well.

The $1n1p$ channel of $\mu^- + ^{28}\text{Si}$ (production of ^{26}Mg) exhibits a significantly large BR compared with other isotopes, although the channel could not be measured in the current experiment. Previous measurements indicated that more than 15(2)% charged-particle emission for natural abundance silicon [43], with prompt γ -ray measurements showing a lower limit of 8.4% [42]. This high BR is also supported by the model calculations employing the preequilibrium emission and the statistical evaporation models, potentially resulting from the low threshold energy required for one-proton emission from ^{28}Al . The calculated results are detailed in Sec. VI C and Fig. 11.

For the charged-particle emission probabilities, Wyttenbach *et al.* [36] reported systematics that the probability of each charged-particle emission channel decreased exponentially with increasing Coulomb barrier. Based on these findings, BRs of $0n1p$, $1n1p$, $2n1p$, $3n1p$, and α channels of silicon isotopes were estimated as approximately 0.7, 4.1, 2.8, 2.8, and 0.9%, respectively. The results obtained for silicon isotopes in this study did not align with the systematics, although the results of ^{27}Al were comparable. This discrepancy may be attributed to the fact that most of the reported targets in the previous research were odd- Z and even- N nuclei, and the particle emission probability depends on the proton number and isotopes, as discussed in this section.

The BRs corresponding to infrequent particle emission channels have been observed and summarized in Table XI. These BRs are too large to be understood from the high-energy component of the excitation function when considering the other particle BRs. In particular, the $4n4p$ and $6n4p$ channels observed with silicon isotopes, which can be assumed as two α and two- α two-neutron emission, cannot be explained without a mechanism that enhances the emission of α particles, such as a cluster structure in the excited state. Notably, the observed nuclei are $^{20,21}\text{F}$, which were successfully measured owing to their high sensitivity to the in-beam activation method. Systematic measurements, including infrequent channels, are essential for further discussions.

C. Comparison with theoretical calculations

The experimental results were compared with two model calculations, PHITS [32] and the MEM calculation [6]. In Figs. 10 and 11, the experimental values, including the present result of the absolute BRs and the previous result of the absolute and lower limit of the BRs summarized in Tables VIII–X, were compared with the model calculations.

PHITS is a general-purpose Monte Carlo particle transport simulation code that utilizes various nuclear reaction models and nuclear data libraries. Muon interaction model was also incorporated in PHITS [77]. The excitation function following the muon nuclear capture process is described using a phenomenological function proposed by Singer [50], with the momentum distribution of nucleon in the nucleus derived by Amado [78]. After the neutron acquires energy, the energy dissipation in the nucleus is calculated using the Jaeri quantum molecular dynamics (JQMD) [79,80]. Subsequently, the evaporation process was determined utilizing the generalized evaporation model (GEM) [81]. To account for the surface effect, which enhances the emission probabilities of light complex particles such as deuteron, triton, and α particles, the surface coalescence model (SCM) [82] was employed. The SCM model was included to reproduce the energy distribution of emitted charged particles, particularly in high-energy regions [4]. Since the default parameters in the PHITS code were different from the originally proposed values, modifications were made in this study, with the SCM parameters $h_0 = 0.26 \text{ GeV fm}/c$ and $D = 2.3 \text{ fm}$ [82]. These parameters define the emission probabilities of light complex particles in the SCM model. The root-mean-square charge radius parameters were set as $R_0 = 3.0610, 3.1224, 3.1176$, and 3.1336 fm for ^{27}Al and $^{28,29,30}\text{Si}$ [83], respectively. The components of the direct and preequilibrium emissions are also represented with shade for PHITS calculation in Fig. 10.

The PHITS calculation reproduced the overall trend of the BR: predominantly emitting neutrons and emitting smaller charged particles and especially accurately reproduced the BR of charged-particle emissions. When focusing on the BR of the $0n0p$ channel, we observed that the $0n0p$ channels of $^{28,30}\text{Si}$ were slightly underestimated, whereas those of ^{27}Al and ^{29}Si were overestimated. In the $0n0p$ channel, the energy peaks in the excitation function are crucial in the result, as discussed in Sec. VI B. The excitation function in the PHITS calculation lacks detailed structures and does not differ among isotopes. Therefore, the ratio of BRs of the $0n0p$ channels of PHITS results was aligned with the threshold energy levels of particle emission for each isotope, inconsistently with the experimental result.

With a closer examination of the BR of neutron emission channels, $1n0p$ channel were underestimated for all isotopes (though ^{28}Si was measured as the lower limit), and the $2n0p$ channel was overestimated for ^{27}Al and ^{30}Si . The discrepancy in neutron emission channels can be attributed to the distribution of the excitation function and characteristics of the particle emission process. To investigate the effect of the latter, the particle emission from each process in the PHITS calculation was examined in terms of neutron multiplicities. Figure 12 shows the average multiplicity of neutrons for

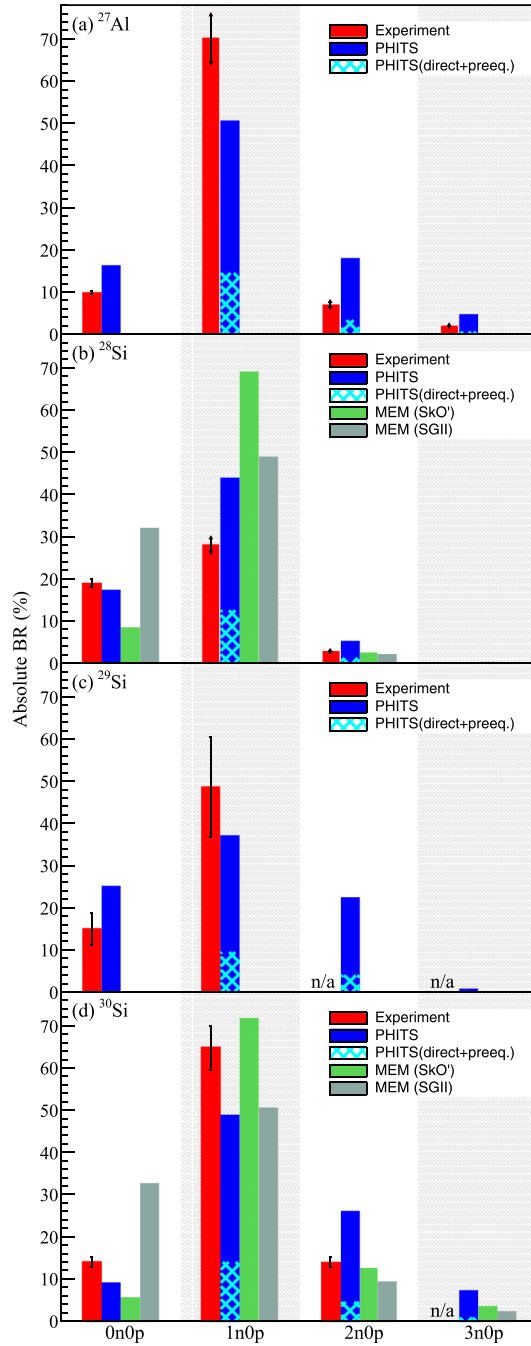


FIG. 10. Comparison of the BR of neutron emission channels between the experimental results and theoretical calculations. The experimental results (in red), the PHITS calculation (in blue), and the MEM calculation using SkO' (in green) and SGII (in gray-green) are presented from left to right for each channel. The experimental results are values listed in Table VIII, including the present and previous results. The experimental values with upper arrow error bars represent lower limits. Channels that are not sensitive to the in-beam activation method and for which no previous measurements exist are labeled as n/a. The components of the direct and preequilibrium are represented with shading for PHITS.

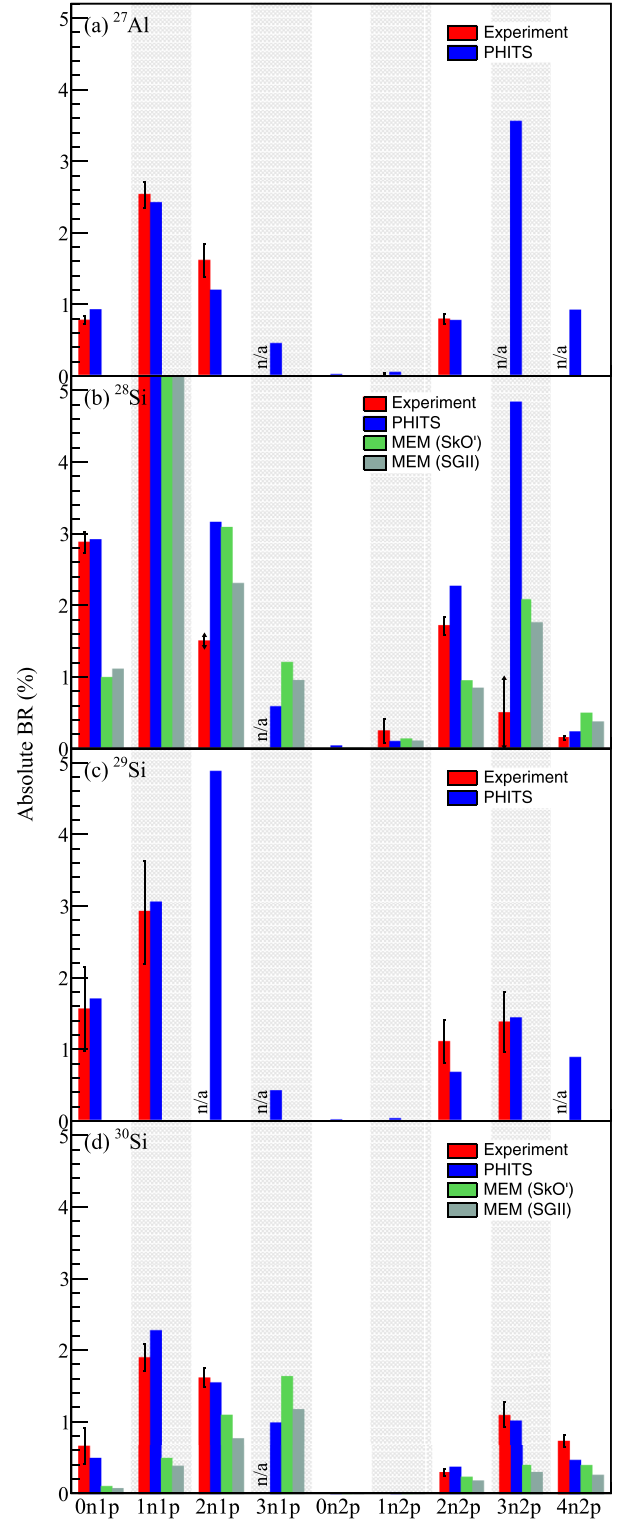


FIG. 11. Same as Fig. 10 for charged-particle emission channels. The experimental results are taken from the values in Tables IX–X. The BR of the 1n1p channel of $\mu^- + ^{28}\text{Si}$ exceeds the range $>8.4(8)$ for the experimental value, 17% for PHITS, and 9.91% and 8.50% for MEM calculation with SkO' and SGII, respectively.

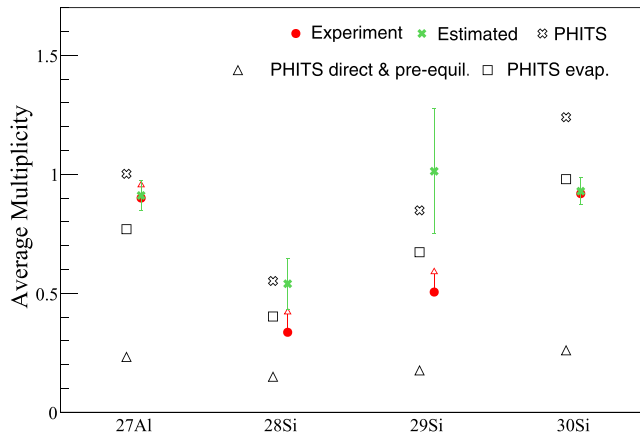


FIG. 12. Average multiplicity of neutron emission, calculated using the $xn0p$ ($x = 1, 2, 3$) channels. The multiplicities, calculated using PHITS, for direct and preequilibrium emissions, were represented by open triangle symbols, whereas those for the evaporation process were denoted by open square symbols. The sum of these processes is indicated using open cross symbols. Experimental results, listed in Table VIII, are represented by filled symbols. If the absolute BR of certain channels was not measured, then the multiplicities are denoted by an upper arrow, indicating a lower limit. The estimated average multiplicities, evaluated using the experimental values, are represented with red-cross-filled symbols.

experimental results, estimation based on the experimental results, and PHITS, calculated with the sum of $xn0p$ ($x = 1, 2, 3$) channels weighted by the absolute BR and the emitted neutron numbers. Only the $xn0p$ channels were adopted here for comparing with the present result. If the absolute BR of certain channels was not measured, then the multiplicities are denoted by an upper arrow, indicating the lower limit. “The estimated average multiplicities” derived from experimental values are also denoted by red-cross-filled symbols. The estimated average multiplicity of ^{28}Si was determined by utilizing the measured neutron multiplicity [49], calculating the contribution from the total $xn0p$ channel of the PHITS calculation. For other isotopes, the average multiplicity was calculated by distributing the BR among unmeasured channels until the total BR of the $xn0p$ channel matched that of PHITS. For example, the $2n0p$ and $3n0p$ channels of ^{29}Si were estimated to have approximately 10% BR, resulting in a total BR of $0n0p$ – $3n0p$ channels of 86.4%, consistent with the total BR calculated by PHITS. The ambiguity of the distributed BR was included in the uncertainty. The average multiplicity from the direct and preequilibrium emissions, calculated during the JQMD, was represented with open square symbols, whereas that of the evaporation process, calculated during the GEM, was represented with open triangle symbols. The summed multiplicity of these processes is shown with open cross symbols.

Focusing on the neutron multiplicity of PHITS, the direct and preequilibrium emission were relatively constant across all isotopes, whereas the average multiplicity from the compound nuclei increased with the neutron excess, $(A - Z)/A$. Since the evaporation process is dominant for multiple neutron emission channels, neutron excess depen-

dence on compound emission leads to a higher probability of multiple neutron emission with increasing neutron excess. This neutron multiplicity dependence on the neutron excess is roughly comparable with both experimental and estimated values. When comparing the estimated value with the summed multiplicity of PHITS results for each isotope, PHITS overestimated the average multiplicity for ^{27}Al and ^{30}Si . Regarding the direct and preequilibrium emission, the emitted neutron energy distribution has been measured for $^{\text{nat}}\text{Si}$ in the previous research [48], reporting 19(3)% of emissions in energies higher than 10 MeV; in contrast, PHITS obtained a lower percentage of 11%. The underestimation of high-energy neutron emission was also reported in the previous measurement of neutron energy spectrum on palladium isotopes [84]. This underestimation in high-energy emission indicates that PHITS underestimated the number of direct and preequilibrium emissions, considering the predominance of those processes in the high-energy neutron emission. Assuming the excitation energy remains constant across isotopes, the direct and preequilibrium emission probabilities do not differ among $^{28,29,30}\text{Si}$ and ^{27}Al . Upon this assumption, the underestimation of direct and preequilibrium emission for $^{\text{nat}}\text{Si}$ suggests that PHITS consistently underestimated the preequilibrium emission for $^{28,29,30}\text{Si}$ and ^{27}Al . Given that preequilibrium emission carries a significant amount of excitation energy, a lower preequilibrium emission could potentially keep the energy distribution at the beginning of the evaporation calculation high. This makes an overestimation of the number of neutrons emitted from the compound nucleus, consistent with the observed overestimation in ^{27}Al and ^{30}Si . The impact of the overestimating multiplicity owing to the lack of the preequilibrium emission became more apparent when the $2n$ and $3n$ channels are more prominent. This could explain why overestimation of multiplicity was observed in ^{27}Al and ^{30}Si , with larger neutron excess, but not in ^{28}Si . When the direct and preequilibrium emission increased, the $1n0p$ channel increased owing to a significant portion of direct and preequilibrium emission belonging to the $1n0p$ channel, as shown in Fig. 10. Additionally, the reduction of higher multiple neutron emission from the evaporation process further contributes to enhancing the $1n0p$ channel. The underestimation of preequilibrium emission may be attributed to low excitation energy or the property of the JQMD model.

For charged-particle emissions, the calculated BRs are consistent with those of the present results, although there are a few exceptions. The summed BRs of the proton emission channels ($xn1p$, $x = 0$ –4) and that of α emission channels ($xn2p$, $x = 2, 3, 4$) are listed in Table XII. These summed BRs of PHITS were comparable with the experimental results. However, by considering an overestimation of the charged-particle emission on palladium isotopes [1] and the discrepancy in neutron emission probabilities, improvements are necessary for PHITS, despite showing consistent values on the BR of charged-particle emission in the mass region of aluminum and silicon.

Recently, MEM has been developed to provide a comprehensive description of the muon nuclear capture process. This model calculates nuclear excitation resulting from the capture process using the second Tamm-Dankoff approxima-

TABLE XII. Sum of the BRs (in percentage) of charged-particle emission channels. The sums of the $xn1p$ ($x = 0-4$) and $xn2p$ ($x = 2-4$) channels are listed for the measured BRs and BRs calculated using PHITS and MEM (SkO' and SGII). The experimental values cover the channels indicated in the column x .

Channel	Target	Expt.		Calc.		
		x		PHITS	SkO'	SGII
$xn1p$ ($x = 0-4$)	^{27}Al	0-2,4	4.97(7)	4.99	—	—
	^{28}Si	0-2	>12.8 (8)	23.9	15.2	12.9
	^{29}Si	0, 1	4.5(12)	10.2	—	—
	^{30}Si	0-2	4.17(43)	5.48	3.32	2.40
$xn2p$ ($x = 2-4$)	^{27}Al	2	0.80(6)	5.54	—	—
	^{28}Si	2-4	>2.4 (5)	7.59	3.51	2.97
	^{29}Si	2, 3	2.5(7)	3.20	—	—
	^{30}Si	2-4	2.11(24)	2.08	1.01	0.72

tion [85,86], with the treatment of 2p-2h model space. In this study, two effective interactions, SkO' [87] and SGII [88], which are often used to describe charge exchange reactions, were employed. The initial state of the nucleus and the muon were calculated using the Skyrin-Hartree-Fock method [89], assuming spherical symmetry and the density functional theory, respectively. Furthermore, the two-body meson-exchange current was included phenomenologically to introduce a high-energy component to the excitation function. Particle emissions from the preequilibrium states were estimated using a two-component exciton model [90,91], whereas emissions from the compound states were computed using a Hauser-Feshbach statistical model [92]. The MEM calculation well reproduced the energy distribution of emitted particles following muon nuclear capture reaction in high-energy regions above 5–10 MeV for ^{28}Si and ^{40}Ca [6]. This calculation can be applied to even-even nuclei, allowing for a comparison of experimental results and calculations for $^{28,30}\text{Si}$ in this study.

The MEM calculation reproduced the general trend of predominantly emitting neutrons, particularly the $1n0p$ channel, and emitting a smaller amount of charged particles. The order of the BR for both the total neutron emission and charged-particle emission channels is also consistent. However, the absolute BRs of each channel showed discrepancies with the experimental results. Regarding the neutron emission channel, the $0n0p$ channel was underestimated with SkO' and overestimated with SGII for both $^{28,30}\text{Si}$. The $0n0p$ channel is sensitive to the energy peak in the excitation energy and the neutron separation energy. The calculated peak energy depends on effective forces, SkO' and SGII, as shown in Fig. 6, and therefore the BR significantly depends on the model utilized in the calculation. Additionally, the MEM calculation did not consider the pn correlation in the final state, which could impact the BR of the $0n0p$ channel, as discussed in Sec. VI B. The measured absolute BRs of the charged-particle emission channels exceeded the calculated values, as listed in Table XII. When comparing the BRs of the $2p$ channel of $\mu^- + ^{28}\text{Si}$ and the $1p$ channel of $\mu^- + ^{30}\text{Si}$, the MEM calculation predicts a higher BR for channels involving higher neutron-accompanied charged-particle emissions, such

as the $3n1p$ channel. However, this fact is inconsistent with the experimental results. The calculation seems to overestimate the neutron emission probability accompanied by charged-particle emission. Given that the high-energy component of the excitation function influenced the emission of multiple particles, the inclusion of meson-exchange current could impact these emission channels. Additionally, the neglected nuclear deformation effect could contribute to discrepancies in production BRs. While the MEM calculation accurately reproduces the energy distribution of emitted charged particles in high-energy regions, a significant discrepancy was observed in the production BRs. Further model refinement is necessary to reproduce the experimental results.

D. Evaluating excitation energy

The absolute BR provides information regarding the excitation energy distribution following the muon nuclear capture reaction. As the primary particle emission results from the evaporation process, the excitation function can be estimated from the production BR, using the statistical evaporation model. The structures of the excitation function of $^{28,30}\text{Si}$ were estimated based on the results of the current measurement.

The production BR of residual nuclei i , b_i , can be expressed with the excitation function populated by muon nuclear capture, $\omega(E^*)$, as

$$b_i = \int_0^\infty P_i(E^*)\omega(E^*)dE^*, \quad (8)$$

where $P_i(E^*)$ represents the probability of the creation of a residual nuclei i with a certain excitation energy at E^* . $P_i(E^*)$ was calculated using the Hauser-Feshbach statistical model implemented in CCONE [91,93]. The initial angular momentum distribution of the excitation function was estimated based on the known average angular momentum distribution, which was calculated from the spin-parity-dependent level density. The excitation function $\omega(E^*)$ was assumed to be expressed as a linear combination of the Gaussian function $G(E^*; E_j, \sigma_j)$ as

$$\omega(E^*) = \sum_j^H c_j G(E^*; E_j, \sigma_j), \quad (9)$$

where c_j represents the coefficient of each Gaussian function. E_j , σ_j , and H represent the mean energy, standard deviation, and number of Gaussian functions with different mean energies, respectively. Substituting Eq. (9) into Eq. (8) results in a χ^2 of the production BR (b_i) and experimental production BR ($b_{\text{exp},i}$), expressed as

$$\chi^2 = \sum_i^N \left(\frac{b_{\text{exp},i} - \sum_j^H c_j \int P_i(E^*)G(E^*; E_j, \sigma_j)dE^*}{\Delta b_{\text{exp},i}} \right)^2, \quad (10)$$

where N represents the number of experimental data and $\Delta b_{\text{exp},i}$ represents an uncertainty of the experiment. The optimum excitation function is determined by deriving c_j to minimize χ^2 . Bayesian optimization with the Gaussian process was employed to deduce the optimum excitation function while constraining the range of $\omega(E^*)$ to avoid unphysical outcomes, namely $\omega(E^*) < 0$. The radius basis function was

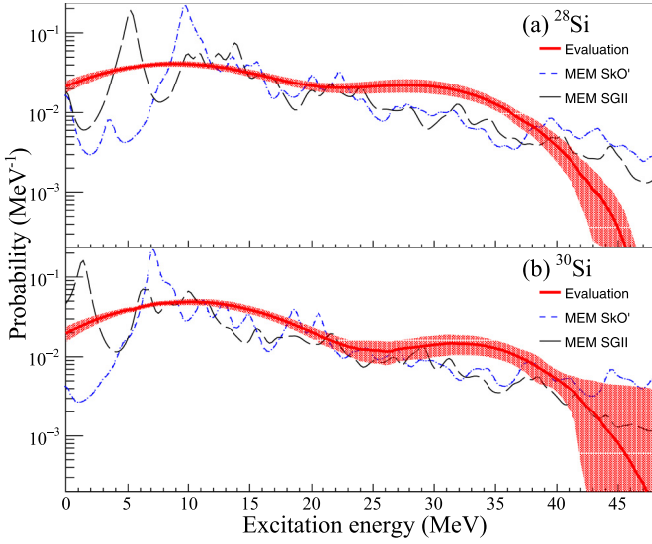


FIG. 13. Estimated excitation function from the experimentally measured BR. The red shaded area represents the uncertainty calculated based on the uncertainty in the experimental BRs. The theoretically calculated excitation function using MEM is represented by black and blue dotted lines.

utilized as the kernel function, with hyperparameters set at 3 and 1 for length and variance, respectively. $H = 20$, $E_j = 2.5 \text{ MeV} \times j$, $\sigma_j = 5.0 \text{ MeV}$ were adopted. These parameters were selected to minimize χ^2 . The total area of the excitation function was normalized to the sum of experimental BRs.

The experimental data utilized in this study included the measured production BR of $^{28,30}\text{Si}$, supplemented by the evaluated production BR for ^{28}Si . The decision to incorporate by the evaluated production BR for ^{28}Si resulted from the limitations in the measured BR in our study, particularly in the dominant component ($1n0p$ and $2n0p$ channels) which could not be measured with the activation method. Previous studies involving prompt γ -ray measurement [42], total charged-particle emission probability [43], and neutron multiplicity measurement [49] were incorporated into the evaluation process, combined with the present result. The estimated values of BRs were as follows: 50(10)% of $1n0p$, 13.5(60)% of $2n0p$, 10.8(30)% of $1n1p$, 1.5(5)% of $2n1p$, 0.5(5)% of $3n2p$, and 0.15(3)% of $4n2p$ channels.

Figure 13(a) and 13(b) show the evaluated excitation functions of ^{28}Si and ^{30}Si , respectively, with the theoretically calculated excitation function using MEM represented by black and blue dotted lines. The evaluated excitation functions based on the experimental data provided insights into the general shape of the excitation function, particularly in the region below approximately 40 MeV, where the estimation by BR is effective. The average excitation energies of ^{28}Si and ^{30}Si are 16.4(3) and 15.4(5) MeV, respectively, and the distributions are exponentially decreasing above 35–40 MeV. The evaluated average excitation energies are slightly larger than those calculated by MEM with SkO' and SGII, which yield 13.8 and 15.6 for ^{28}Si and 11.1 and 14.7 for ^{30}Si , respectively. At higher energies, direct and preequilibrium particle emission mechanisms come into play, so this discussion alone cannot

provide conclusive evidence. The present proposed excitation functions do not exhibit detailed structures of transition peaks because the excitation energy information included in the production BRs is limited by the energy resolution determined by the gaps between successive threshold energies. Further inclusion of the experimental results of direct feeding levels, obtained using the prompt γ -ray measurement in coincidence with emitted particles, into the evaluation input is expected to enable providing the peak structures between each threshold energy. Evaluation of the initial angular momentum distribution and consideration of other observables, such as the energies of emitted particles, are also necessary. However, the method of deriving the excitation function from measured production BR was demonstrated.

VII. SUMMARY AND CONCLUSIONS

The absolute production BRs of residual nuclei following muon nuclear capture of ^{27}Al , $^{\text{nat}}\text{Si}$, and $^{28,29,30}\text{Si}$ were measured at two pulsed muon beam facilities: RIKEN-RAL and MLF, J-PARC. The experiments employed the in-beam activation method, utilizing a plastic scintillator for muon counting and high-purity germanium detectors for measuring β -delayed γ rays emitted by the produced nuclei. Absolute BRs were measured using a countable intensity muon beam at RAL, and a high-statistic BR measurement using a high-intensity pulsed muon beam was conducted at J-PARC. The number of muons irradiating the target was calibrated for the J-PARC experiment to determine the absolute BRs. The results for $^{28,29,30}\text{Si}$ were derived through a decomposition analysis utilizing data from the $^{\text{nat}}\text{Si}$ target.

The measured BRs were discussed, focusing on the excitation function, particle emission mechanism following the muon nuclear capture reaction, and nuclear properties. The results revealed that neutron emission without charged-particle emission was the predominant process for all target isotopes, with the $1n0p$ channel demonstrating the highest BR. The production BRs also suggested an average excitation function peaking between 10 and 20 MeV, with evidence of excitation energies exceeding 30 MeV. Comparison of the $0n0p$ channel between ^{27}Al and $^{28,29,30}\text{Si}$ revealed a significantly smaller $0n0p$ channel for ^{27}Al . This trend of higher neutron emission probabilities in odd-Z nuclei was also found in previous measurements. Small BRs in the $0n0p$ channel of muon nuclear capture of odd-Z nuclei were discussed with the evaluation of the GT strength below the one neutron separation energy, emphasizing the importance of considering the final interaction of proton-neutron pairs for the muon nuclear capture reaction. This indicates the potential of using muon nuclear capture as a probe to investigate isovector transitions and nucleon pairing effects. For the charged-particle emission channels, the measured BRs for the $0n1p$ and $2n2p$ channels decreased as neutron excess increased. Additionally, the neutron emission, coupled with charged-particle emissions, demonstrated an increase with neutron excess, indicating a correlation between neutron excess and the probability of charged-particle emission. The observation of BRs for rare particle emission channels, such as the $4n4p$ and $6n4p$ channels, indicated a potential mechanism that enhances the emission of α particles,

such as cluster structures in the excited state. These results of the BR of neutron and charged-particle emission channels indicate that the even-odd effect of neutron and proton numbers, as well as neutron excess, influence particle emission following muon nuclear capture.

The experimental results were compared with previous measurements and theoretical model calculations. The present results were generally consistent with previous prompt γ -ray measurements and direct particle detection experiments. This study provided the most accurate absolute BR for ^{27}Al , ^{28}Si , and $^{29,30}\text{Si}$. By comparing the $0n0p$ channel obtained through prompt γ -ray measurements with the results of this study, over 88% of the $0n0p$ channel was covered by the prompt γ -ray measurement either in ^{27}Al and ^{28}Si , suggesting a few ground-state transitions in the $0n0p$ channels. Comparisons with theoretical model calculations using PHITS and MEM revealed both agreements and discrepancies. PHITS reproduced overall trends but overestimated neutron multiplicities and the $0n0p$ channel for some isotopes. MEM reproduced the general order of BRs for total neutron and charged-particle emission channels but underestimated charged-particle emission BRs. Notably, a significant model dependence was observed, particularly in neutron emission channels. Even models that accurately reproduced the energy distribution of neutrons and charged particles struggled to accurately represent the production BRs. This indicates the necessity of investigating muon nuclear capture, including BR measurements. Additionally, a method for estimating the excitation function from the measured production BR was proposed utilizing the theoretical model of statistical evaporation emission.

To delve deeper into the origin and characteristics of observed features of particle emissions, systematic measurements considering the even-odd effect and neutron excess, and additional experiments, such as direct charged-particle measurement coinciding with neutron measurement, are imperative. The relative ease of measuring production BRs presents

an advantage for conducting systematic measurements. Further theoretical refinements are necessary to accurately predict the observed BRs, which would enhance our comprehensive understanding of weak response in nuclei and the particle emission mechanism from the highly excited states.

In conclusion, this study highlights the significance of considering nuclear structure effects, even-odd effects of neutron and proton numbers, neutron excess, and different particle emission mechanisms to effectively model this complex reaction. This study also demonstrated the potential of production BR measurement in extracting properties of excitation transitions and reaction dynamics of muon nuclear capture.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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