# DC Magnetic Field-Based Analytical Localization Robust to Known Stationary Magnetic Object

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***Abstract*—Indoor localization technologies are attracting attention since those are required by navigation systems, motion tracking, IoT, etc. However, currently available indoor localization methods suffer from high installation costs and insufficient robustness. Among them, DC magnetic field-based localization is a promising method for indoor localization since it requires only a single anchor coil and achieves low installation cost. On the other hand, the DC magnetic field is seriously distorted by the magnetic objects placed in the localization environment, which degrades localization accuracy.**

**This work proposes a DC magnetic field-based localization method robust to known stationary magnetic objects. The proposed method formulates and solves the simultaneous equations for localization that consider the magnetic fields generated by both the anchor coil and magnetic object, where the equations are derived from a magnetic field analysis method called equivalent current method. Simulation results show that the proposed method localizes the sensor position with the maximum error of 6.13 mm in a 200 mm $\times$ 300 mm area when the sensor is placed more than 50 mm away from the magnetic object. Even when the sensor is placed at a distance of 50 mm away from the magnetic object, the proposed method can achieve the maximum localization error of 16.58 mm, which is 50 % smaller than that of the conventional method.**

***Index Terms*—Indoor localization, coil, DC magnetic field, Magnetic Object, Internet of Things (IoT)**

## I. Introduction

The importance of localization technology is increasing in the era of IoT, since the value of the information gathered by IoT devices elevates greatly when it is associated with the location information. Here, the localization technology can be classified into two types: outdoor and indoor localization. For outdoor localization, GPS-based methods are widely used since those can achieve accurate real-time localization. On the other hand, when it comes to indoor, GPS-based methods cannot provide sufficient accuracy since the GPS signal severely attenuates indoors.

To address this issue, many indoor localization methods using other signal sources than GPS have been proposed. Currently-available localization methods include the following four major methods: camera-based method, Wi-Fi-based method, AC and DC magnetic field-based methods. Let us review them briefly. Camera-based methods can achieve high localization accuracy by installing only one camera, which greatly contributes to the low installation cost, while privacy and occlusion issues exist [1]. Similarly, the installation cost of Wi-Fi-based methods is also low. However, to operate a practical localization system, a Wi-Fi site survey is required, and hence the operation cost is high [2]. Magnetic field-based methods are expected to be low-cost and accurate localization methods [3], [4]. Both AC and DC magnetic field-based methods estimate the sensor position with the magnetic flux density generated by anchor coils. AC magnetic field-based method can provide a wider localization area than DC one since currently available AC magnetic field sensors are more sensitive than DC ones and can detect subtle magnetic field change. However, AC magnetic field-based method has a robustness problem since the AC magnetic field is seriously distorted by metallic objects in the localization area [5]. In addition, most of the AC magnetic field-based methods require multiple anchor coils, which results in the high installation cost. In contrast, DC magnetic field-based method is more robust than AC one since DC magnetic field is not affected by nonmagnetic materials. Ref. [4] proposes a DC magnetic field-based localization method with a single anchor coil achieving low installation cost. However, Ref. [4] points out that the magnetic objects placed in the localization area cause the localization accuracy degradation. For this reason, a DC magnetic field-based localization method that is robust to magnetic objects is highly demanded.

This paper proposes a DC magnetic field-based analytical localization method robust to known stationary magnetic material. The proposed method analytically calculates the effect of the magnetic material with "Equivalent Current Method (ECM)" proposed in Ref. [6], and localizes the sensor position with a single anchor coil.

## II. Analysis of DC Magnetic Field

This section introduces analysis methods of artificially generated DC magnetic field. Section II-A analyzes the DC magnetic field generated by a single loop coil, which is utilized in the localization method proposed in Ref. [4]. Here, the localization error of the method proposed in Ref. [4] elevates significantly if a magnetic object is placed in the localization system. Therefore, successive Section II-B introduces the method which can calculate the magnetic field affected by magnetic material. Also, Section II-B discusses

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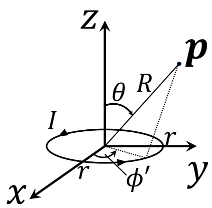


Fig. 1. Coordinate setting.

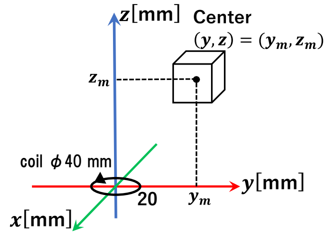


Fig. 2. System containing a magnetic object.

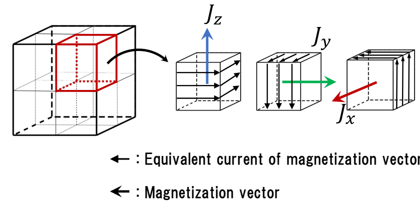


Fig. 3. *Equivalent current method* approximates the magnetization vectors with equivalent surface current flow.

the applicability of the method to the localization robust to the magnetic object.

### A. Magnetic Field Generated by Coil

The localization method proposed in Ref. [4] localizes the sensor position based on the magnetic flux density vector generated by the anchor coil. For this reason, this section introduces the analysis method of the DC magnetic field generated by the coil. Fig. 1 shows the coordinate setting and definition of the point $\boldsymbol{p}(R,\theta,\phi')$ and the radius of a single-loop anchor coil $r$. The DC magnetic field at the point $\boldsymbol{p}$ generated by the anchor coil is expressed as follows [7]:

$$\boldsymbol{B_c}(\boldsymbol{p}) = \nabla \times \boldsymbol{e}_{\phi'} \frac{\mu_0 I r}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin\phi}{\sqrt{R^2 + r^2 - 2rR\sin\theta\sin\phi}} d\phi, \tag{1}$$

where $\boldsymbol{e}_{\phi'}$ is a unit vector pointing $\phi'$ direction, $\phi$ is an integral variable, and $\mu_0$ is a magnetic constant. The magnetic field generated by a solenoid whose winding number is larger than one can be calculated by superposing the analysis results of multiple single loop coils.

### B. Magnetic Field Affected by Magnetic Object

This section derives the magnetic field affected by the magnetic object. Fig. 2 illustrates the DC magnetic field-based localization system containing a magnetic object. The DC magnetic field existing in the system shown in Fig. 2 consists of two components: DC magnetic field generated by the current flow in the anchor coil and by the magnetization of the magnetic object. The former can be analyzed by the equation introduced in Section II-A with low computational cost. However, the latter requires high computational cost since the the magnetization strongly and complexly depends on the exposed magnetic field. Conventionally, the effect of the magnetization is analyzed with Finite Element Method (FEM), which derives magnetic field with high accuracy while requiring huge computation [8]. The massive computation prevents us from building a real-time localization method, and hence the method that can analyze the effect of the magnetization with low cost is highly demanded.

To address this issue, this work focuses on Equivalent current Method (ECM) proposed in Ref. [6], which enables us to calculate the effect of the magnetization with low calculation cost. ECM divides the magnetic object shown in Fig. 2 into $n$ equal elements and derives magnetization vectors $\boldsymbol{J} = (J_x, J_y, J_z)$ for each element. ECM approximates each of $J_x, J_y$ and $J_z$ with an equivalent surface current as illustrated in Fig. 3. By taking advantage of this approximation, we can calculate the magnetization-induced magnetic field with Biot-Savart Law. Consequently, we can represent the magnetic flux density $\boldsymbol{B}(\boldsymbol{p})$ at a point $\boldsymbol{p}$ with Eq. (2):

$$\boldsymbol{B}(\boldsymbol{p}) = \boldsymbol{M}(\boldsymbol{p}) \cdot \boldsymbol{J} + \boldsymbol{B_c}(\boldsymbol{p}), \tag{2}$$

where $\boldsymbol{M}(\boldsymbol{p})$ is the coefficient matrix, which is defined in Ref. [6], for deriving the magnetic flux density generated by the magnetization vector $\boldsymbol{J}$. $\boldsymbol{B_c}(\boldsymbol{p})$ is the magnetic flux density generated by the coil and is given by Eq. (1). We can derive $\boldsymbol{M}$ and $\boldsymbol{J}$ from the following information of the magnetic object: the center coordinate, the shape, the size, and the relative permeability [6]. Eq. (2) always gives the three equations regarding $x, y,$ and $z$, and the number of equations is independent of the number of elements $n$. In other words, ECM requires only three equations to derive the magnetic flux density, which is much less computationally expensive than the FEM-based method.

To confirm the accuracy of ECM, this work conducts a simulation experiment comparing the result of ECM $\boldsymbol{B}_{\mathrm{ECM}} = (B_{\mathrm{ECM}x}, B_{\mathrm{ECM}y}, B_{\mathrm{ECM}z})$ with that of *Femtet* [9], a FEM-based DC magnetic field simulator, $\boldsymbol{B}_{\mathrm{FEM}} = (B_{\mathrm{FEM}x}, B_{\mathrm{FEM}y}, B_{\mathrm{FEM}z})$. The simulation setup is based on Fig. 2 setting, where the coil radius to 20 [mm] and the coil current to 1 [A]. The size and shape of the magnetic object are set to 50 mm $\times$ 50 mm $\times$ 50 mm cube and the material is assumed to be iron whose relative permeability is 5000. As for the position of the magnetic object, we simulate two cases: $(y_m, z_m)$ = (0, 150) and (100, 150). We calculate the magnetic flux density with both ECM and Femtet on the $yz$-plane in the area of $0 \le y \le 250,\ 0 \le z \le 300$ [mm] with the step of 50 mm. Then we calculate the error between them. The error of ECM $\boldsymbol{B}_{\mathrm{Err}} = (B_{\mathrm{Err}x}, B_{\mathrm{Err}y}, B_{\mathrm{Err}z})$ is defined as follow:

$$\boldsymbol{B}_{\mathrm{Err}} = \sum_{i=x,y,z} \left| \frac{B_{\mathrm{FEM}i} - B_{\mathrm{ECM}i}}{B_{\mathrm{FEM}i}} \right| \cdot \boldsymbol{e}_i, \tag{3}$$

where $\boldsymbol{e}_x, \boldsymbol{e}_x,$ and $\boldsymbol{e}_z$ are unit vectors pointing $x, y,$ and $z$-direction. Here, The $x$-component of the magnetic flux density is zero on the $yz$-plane when the coil is placed as Fig. 2 [7], and hence the following discussion focuses only on $\boldsymbol{B}_{\mathrm{Err}y}$ and $\boldsymbol{B}_{\mathrm{Err}z}$. When the position of the magnetic object is set as $(y_m, z_m) = (0, 150)$ [mm], both $B_{\mathrm{Err}y}$ and $B_{\mathrm{Err}z}$ are less than $10\%$ at points more than 50 mm away from the magnetic object. In this situation, the maximum error is $14.8\%$

of $B_{\mathrm{Err}z}$ at point $(y,z) = (0,200)$. When the position of the magnetic object is set as $(y_m, z_m) = (100, 150)$ [mm], both $B_{\mathrm{Err}y}$ and $B_{\mathrm{Err}z}$ are less than $11.0\%$ at points more than 50 mm away from the magnetic object. In both situations, approximation error becomes smaller as the distance between the measurement point and the magnetic material becomes larger. These results indicate that the ECM can provide a good approximation and is applicable for the analytical localization.

## III. Proposed Localization Method

This section proposes a localization method that is robust to the known stationary magnetic object. The proposed localization method uses ECM, which can calculate the DC magnetic field affected by the magnetization with low computational cost as explained in the previous section. We perform localization with the inverse operation of ECM.

The proposed localization method estimates the position $\boldsymbol{p}$ of the sensor placed in the $yz$-plane with the magnetic flux density measured by the sensor. The supposed magnetic object is a rectangular shape with its center in the $yz$-plane and each side parallel to the $x$, $y$, and $z$-axis. The proposed localization method requires the following four pieces of information: the magnetic flux density $\boldsymbol{B}$ measured by the sensor, the shape of the coil, the current applied to the coil, and the magnetic object information. Here, the information about the magnetic object includes the center coordinates of the magnetic object, the length of each side of the rectangle, the number of elements $n$, and the relative permeability of the magnetic object.

Let us explain the inverse operation of ECM, which is the key idea of the proposed localization method. First, the proposed method derives the magnetization vector $\boldsymbol{J}$ with ECM. The magnetization vector is derived from information of the coil and the magnetic object and represented by Eq. (4):

$$\boldsymbol{J} = (\boldsymbol{J}_1, \cdots, \boldsymbol{J}_i, \cdots, \boldsymbol{J}_n)^{\mathrm{T}}. \tag{4}$$

$\boldsymbol{J}_i (i = 1, 2, \cdots, n)$ is the magnetization vector of the element $i$. $\boldsymbol{J}_i$ can be represented by Eq. (5):

$$\boldsymbol{J}_i = (J_{ix}, J_{iy}, J_{iz}). \tag{5}$$

$J_{ix}, J_{iy}, J_{iz}$ are the values of the magnetization vector in the $x, y,$ and $z$ directions, respectively. If information about the magnetic object and the coil are unchanged, we can reuse the same magnetization vector previously derived. Next, we derive the coefficient matrix necessary to calculate the magnetic flux density generated by the current equivalent to the magnetization vector. The coefficient matrix can be derived from information of the magnetic object and represented by Eq. (6):

$$\boldsymbol{M}(\boldsymbol{p}) = (\boldsymbol{m}_1(\boldsymbol{p}), \cdots, \boldsymbol{m}_i(\boldsymbol{p}), \cdots, \boldsymbol{m}_n(\boldsymbol{p})), \tag{6}$$

where $\boldsymbol{m}_i (i = 1, 2, \cdots, n)$ is the coefficient matrix for deriving the magnetic flux density generated by $\boldsymbol{J}_i$. $\boldsymbol{m}_i$ can be represented as Eq. (7):

$$\boldsymbol{m}_i = \begin{pmatrix} \alpha_{ix}(\boldsymbol{p}) & \beta_{ix}(\boldsymbol{p}) & \gamma_{ix}(\boldsymbol{p}) \\ \alpha_{iy}(\boldsymbol{p}) & \beta_{iy}(\boldsymbol{p}) & \gamma_{iy}(\boldsymbol{p}) \\ \alpha_{iz}(\boldsymbol{p}) & \beta_{iz}(\boldsymbol{p}) & \gamma_{iz}(\boldsymbol{p}) \end{pmatrix}, \tag{7}$$

where $\alpha_{id}, \beta_{id},$ and $\gamma_{id}$ $(d = x, y, z)$ are the coefficients to derive the magnetic flux density generated by $J_{ix}, J_{iy}, J_{iz}$ in the $d$ direction, respectively.

Finally, we build Eq. (8) by substituting the magnetic field $\boldsymbol{B}$ measured by the sensor, Eq. (4), and Eq. (6) for Eq. (2):

$$\boldsymbol{B} = \boldsymbol{M}(\boldsymbol{p}) \cdot \boldsymbol{J} + \boldsymbol{B_c}(\boldsymbol{p}). \tag{8}$$

Eq. (8) has only $\boldsymbol{p}$ as a variable, and hence we can estimate the position of the sensor from the magnetic flux density $\boldsymbol{B}$ measured by the sensor.

## IV. Evaluation

This section conducts simulation evaluations to confirm the effectiveness of the proposed method. Section IV-A explains the constraints and the numerical methods for solving equations. Section IV-B conducts the localization with the proposed method in the system containing the magnetic object and compares the result with the conventional method.

### A. Setup

The proposed localization method estimates the sensor position by solving the simultaneous equation Eq. (8). The calculation complexity of Eq. (6) and Eq. (7) depends on the number of elements into which the magnetic material is divided. To confirm the effectiveness of the proposed method with low computation cost, this work assumes that the division number of the magnetic object is one. In addition, Ref. [4] points out that the magnetic field generated by the coil is axially symmetrical, and hence the feasibility of 3D localization is guaranteed as long as the localization can be performed in one 2D-plane. Therefore, to simplify the discussion, this work performs the 2D localization on the $yz$-plane.

The proposed method has to calculate the DC magnetic field generated by the anchor coil with Eq. (1). Ref. [7] points out that Eq. (1) is approximated by Eq. (9) when $R$ is sufficiently larger than $r$, which is the reasonable assumption in most localization environments. This work also takes advantage of this approximation to further reduce the computation cost.

$$\boldsymbol{B_c} = \frac{\mu_0 I r^2}{2R^3}\cos\theta \boldsymbol{e_r} + \frac{\mu_0 I r^2}{4R^3}\sin\theta \boldsymbol{e_\theta}. \tag{9}$$

To perform localization, a method for deriving the solution of Eq. (8) is required. We can utilize any method as long as it can solve simultaneous equations. In this work, we utilize the *Newton's method* to solve Eq. (8). Newton's method can solve nonlinear simultaneous equations of one or more variables [10] even with the machine whose computing power is limited. As an implementation of Newton's method, we used the "mnewton" function of Maxima [11], which is a mathematical processing software. Newton's method requires the initial guess value to derive the solution. Choosing an inappropriate initial guess value results in diverging from the root and hence the initial value should be carefully set. In the system illustrated in Fig. 2 in which the only one anchor coil is installed, the appropriate initial value depends on whether the sensor is placed around the $z$-axis or not. Therefore, if the direction of the measured magnetic flux density is pointing to $z$-direction, which means the sensor is around the $z$ axis, we choose $(y, z) = (0, \textit{non zero value})$ as an initial value. Otherwise, the system iteratively tries multiple initial values until the output of Newton's method converges.

### *B. Localization Result*

This section evaluates the performance of the proposed localization method in the system illustrated in Fig. 2, which contains a magnetic object same as one discussed in Section II-B. Experiments are conducted under two situations: one with the position of the magnetic object $(y_m, z_m)$ is set to (0, 150) [mm] and the other is (100, 150) [mm]. The coil radius is 20 mm and the current injected to the coil is set to 1A. Here, to eliminate the influence of sensing noise, we utilized the analysis result of Femtet for localization instead of the output of an actual magnetic sensor.

Fig. 4(a) shows the localization result and the comparison with the conventional method [4] when $(y_m, z_m) = (0, 150)$ [mm]. Fig. 4(a) indicates that the localization accuracy is greatly improved with the proposed method compared to the conventional method. In Fig. 4(a), the localization error with the conventional method elevates up to 35.69 mm because of the magnetic object, while the proposed method keeps the error to 16.58mm at the point (50, 150) [mm]. At the points more than 50 mm away from the magnetic object, the localization error is reduced to only 6.13 mm. Fig. 4(a) also indicates that the localization errors of the proposed method at $(y, z) = (50, 150)$ [mm] and $(0, 100)$ [mm] are relatively larger compared to other points. These localization error degradations are because of the approximation accuracy of ECM. The issue on the accuracy of ECM may be improved by increasing the dividing number of the magnetic object from one to plural, which is one of our future works.

Fig. 4(b) shows the localization result and the comparison with the conventional method [4] when $(y_m, z_m) = (100, 150)$ [mm]. Fig. 4(b) indicates that the localization accuracy is improved with the proposed method compared to the conventional method. In Fig. 4(b), the proposed localization method achieves high accuracy with a maximum error of 5.74 mm while that of the conventional method [4] is 54.75 mm.

## V. Conclusion

This paper proposed a DC magnetic field-based localization method that is robust against the known stationary magnetic objects. The proposed method obtained robustness against the magnetic material with low computational cost by using ECM, which approximates the effect of magnetization with equivalent surface current on the magnetic material. Simulation results showed that the proposed method improved the localization error from 35.69mm, which is achieved by the conventional method, to 16.58 mm even when the magnetic material is placed near the sensor node. When the sensor node is placed more than 50 mm away from the magnetic material, the proposed method further reduced the localization error to 6.13 mm. When the magnetic object is placed away from the anchor coil, the proposed method achieves the maximum localization error of 5.74 mm while the conventional method achieves only 54.75 mm.

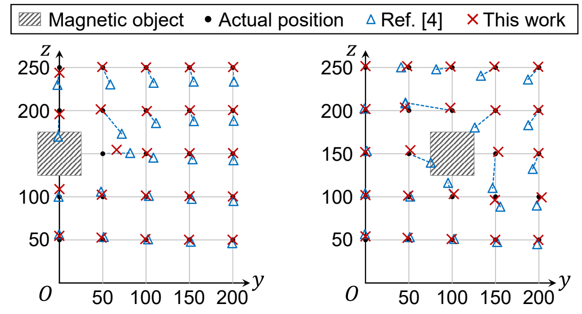


Fig. 4. Localization results and comparisons with conventional method in two situations. (a)$(y_m, z_m) = (0, 150)$ [mm], (b)$(y_m, z_m) = (100, 150)$ [mm]. In both situations, the anchor coil is placed in the $xy$-plane and the center of the coil is set to the origin.

Our primary future work is to minimize the localization error when the magnetic object is placed near the anchor coil. The simulation result showed that the approximation accuracy of the ECM in such a case degrades significantly, and consequently the localization accuracy deteriorates. Therefore, we work on improving the accuracy of ECM to enhance the localization performance.

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